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Dynamic structural modification of vibrating systems oriented to eigenstructure assignment through active control: A concurrent approach

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Eigenvalues and eigenvectors play a fundamental role in determining the dynamic behavior of a vibrating system. Thus, an important goal in vibration control is to modify the eigenstructure to match the design specifications. Feedback control is a popular approach to this problem, which is however not always satisfactory. In fact, while system controllability suffices to assure that any requirement on the eigenvalues can be met by the controlled system, desired closed-loop eigenvectors cannot be attained in general, due to inherent limitations of active control.

To overcome such a problem, this paper proposes a hybrid approach in which the mechanical system and the controller are concurrently designed to improve the attainability of the desired eigenstructure. Indeed, the suitable modification of the system inertial and elastic parameters modifies the set of eigenvectors that can be achieved through active control.

In this work is demonstrated that such an objective can be effectively accomplished by minimizing the rank of a certain matrix which depends on the features of the original system and on the desired eigenpairs. Two algorithms for rank minimization are described and adjusted for the problem under consideration.

The method is validated with three examples. In the first one, eigenstructure assignment of a 5 degrees of freedom system that previously appeared in the literature is performed, demonstrating the effectiveness of the proposed approach with respect to the state of the art. In the second example the same 5 degrees of freedom system is considered by also including damping, to evaluate the method capability to deal with damped systems. The third one is a 30 degrees of freedom system that enables the comparison of the two algorithms for different choices of the actuation.

1. Introduction

The dynamic behavior of vibrating linear systems is expressed by the well-known system of second-order differential equations

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$$\mathbf{M}\ddot{\mathbf{q}}(t)+\mathbf{C}\dot{\mathbf{q}}(t)+\mathbf{K}\mathbf{q}(t)=\mathbf{0}, \tag{1}$$

where $\mathbf{M},\mathbf{C},\mathbf{K}\in\mathbb{R}^{N\times N}$ are respectively the symmetric mass, damping and stiffness matrices, $\mathbf{q}\in\mathbb{R}^N$ is the displacement vector, and N is the number of degrees of freedom. It is also supposed that $\mathbf{M}$ is positive definite and $\mathbf{C},\mathbf{K}$ are positive semi-definite. It is known, from the theory of linear differential equations, that the solution of such an equation is characterized by the eigenvalues $\lambda_i\in\mathbb{C}$ and the eigenvectors $\mathbf{u}_i\in\mathbb{C}^N$ of the system, for $i=1,\ldots,2N$, namely the solutions of the eigenproblem

$$\left[\lambda_i^2\mathbf{M}+\lambda_i\mathbf{C}+\mathbf{K}\right]\mathbf{u}_i=\mathbf{0}. \tag{2}$$

Eigenvalues and eigenvectors, together, constitute the *eigenstructure* of the system. The design of vibrating systems aimed at satisfying specifications on eigenvalues and eigenvectors, which is commonly known as *eigenstructure assignment*, has drawn increasing interest over the recent years. The most natural mathematical framework for such problems is constituted by the inverse eigenproblems, which consist in the determination of the system model that features a desired set of eigenvalues and eigenvectors. Although such a problem is intrinsically challenging, several solutions have been proposed in the literature. The approaches to eigenstructure assignment can be basically divided into passive control and active control.

Passive control, also known as dynamic structural modification (DSM), aims at achieving the desired eigenstructure by modifying the physical properties of the system (typically the inertial and elastic parameters). In practice, the mass and stiffness matrices are altered, usually by additive modification $\Delta\mathbf{M},\Delta\mathbf{K}\in\mathbb{R}^{N\times N}$, in such a way that the modified system

$$(\mathbf{M}+\Delta\mathbf{M})\ddot{\mathbf{q}}(t)+\mathbf{C}\dot{\mathbf{q}}(t)+(\mathbf{K}+\Delta\mathbf{K})\mathbf{q}(t)=\mathbf{0} \tag{3}$$

features the desired eigenstructure. Such an approach is often chosen because it does not require neither electronic devices nor external power. Moreover, the resulting system is assured to be stable, as long as symmetry and positive definiteness are preserved. However, in practice, not every desired specification can be actually obtained, because the modifications must be feasible (for example, the mass of a component is bounded by technical or economic constraints). Therefore, passive control is not always capable of assigning the desired eigenstructure with sufficient accuracy.

Active control consists in employing a feedback scheme in such a way that the closed-loop system features the desired eigenstructure [1–4]. With that goal, suitable external forces are exerted on the system by controlled actuators. For example, let $\mathbf{f}_C\in\mathbb{R}^N$ be the vector that represents such forces, then the equation of motion becomes

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\mathbf{C}\dot{\mathbf{q}}(t)+\mathbf{K}\mathbf{q}(t)=\mathbf{f}_C(t). \tag{4}$$

Although such an approach requires the use of actuators and sensors, it is often more convenient with respect to DSM, for example in the assignment of damped modes or simply whenever the structural modifications are impractical. Within the field of control of structures, the methods that rely on the second-order linear model are of particular interest, because the properties of the matrices $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ (e.g. symmetry or sparsity) can be exploited. One of the earliest examples of this approach is given by Juang and Maghami [5], that propose a robust method for eigenvalue assignment via velocity and displacement feedback or velocity and acceleration feedback. Such a method has been later modified by Chu and Datta in Ref. [6]. A gradient flow approach for robust eigenvalue assignment has been developed by Chan et al. for state feedback [7] and by Ho et al. for output feedback [8]. Partial pole assignment is achieved by Datta et al. for single-input control [9] and by Ram and Elhay for multiple-input control [10]. Robustness in partial eigenvalue assignment is addressed by Qian and Xu in Refs. [11,12]. There are also several recent contributions to the field of eigenvalue assignment, such as the optimization based approaches by Brahma and Datta [13] and by Bai et al. [14].

Assignment of eigenvalues in combination with their respective eigenvectors has also been addressed in numerous papers. For example, Schulz and Inman in Ref. [15] derive a parametrization of the feedback gain matrices and they exploit it to optimize different design criteria. Triller and Kammer use a control coordinate system based on the Craig-Bampton substructure representation [16]. Kim et al. developed a method based on the solution of the Sylvester equation [17]. Datta et al. achieve partial eigenstructure assignment by appropriate choice of gain and input influence matrices [18]. Nichols and Kautsky tackle the robust eigenstructure assignment problem for second-order systems by solving the generalized linear problem subject to structured perturbations [16]. Duan and Liu proposed a parametric method which uses proportional-plus-derivative feedback in Ref. [19]. Zhang et al. achieved partial eigenstructure assignment method through acceleration and displacement feedback [20]. As an alternative to the second-order representation of the system, the receptance-based formulation has also become popular in the eigenstructure assignment literature [21,22]. For example, systems with time delay have been considered by Bai et al. in Ref. [23], where both receptance and system matrices are used.

In most of the reviewed papers, attention is paid to eigenvalues while eigenvectors are often neglected, since eigenvector assignment is a more challenging issue. In fact, it can be proven that active control can assign only eigenvectors that belong to certain vector spaces that depend on the physical properties of the system and on the characteristics of its actuation systems. The dimension of such a space of allowable eigenvectors matches the number of independent actuation forces. Thus, if the system is highly underactuated it is very unlikely that exact assignment of eigenvectors can be accomplished.