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The modal density of composite beams incorporating the effects of shear deformation and rotary inertia

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A R T I C L E   I N F O

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A B S T R A C T

Engineers and designers are often faced with the task of selecting materials that minimizes structural weight whilst meeting the required strength and stiffness. In many cases fibre reinforced composites (FRCs) are the materials of choice since they possess a combination of high strength and low density. Depending on the application, composites are frequently constructed to form long slender beam-like structures or flat thin plate-like structures. Such structures when subjected to random excitation have the potential to excite higher order vibratory modes which can contribute significantly to structure-borne sound. Statistical Energy Analysis (SEA) is a framework for modeling the high frequency vibration of structures. The modal density, which is typically defined as the number of modes per unit Hertz in a frequency band, is a fundamental parameter when applying SEA. This study derives formulas for the modal density of a fibre reinforced composite beam coupled in bending and torsion. The effects of shear deformation and rotary inertia are accounted for in the formulation. The modal density is shown to be insensitive to boundary conditions. Numerical analyses were carried out to investigate the variation of modal density with fibre orientation including and excluding the effects of shear deformation and rotary inertia. It was observed that neglecting such effects leads to underestimating the mode count in a particular frequency band. In each frequency band there exists a fibre orientation for which the modal density is minimized. This angular orientation is shown to be dependent on the shear rigidity as well as the bending, torsional and coupling rigidities. The foregoing observation becomes more pronounced with increasing frequency. The paper also addresses the modal density beyond the wave-mode transition frequency where the beam supports three propagating waves.

1. Introduction

Fibre reinforced composites represent a class of materials that can be tailored to possess the strength and stiffness of conventional metals but with a fraction of the weight. Consequently, such materials are being utilized in industries where weight is an important design consideration. The dynamic behaviour of composites is dependent on the orientation of fibres within the matrix material. Due to the directional nature of the fibres, composites possess structural anisotropy; a feature that is in sharp contrast to isotropic metals. In particular, anisotropy in slender structures has been known to result in the coupling of bending and torsional degrees of freedom [1]. Several studies have investigated the free and forced vibration characteristics of beams coupled in bending and torsion. Refs. [2,3] experimentally determined the lower order natural frequencies and mode shapes of composite cantilevered beams. Ref. [4] derived the equations of motion of a composite beam and included the effects of rotary

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Nomenclature

A	cross-sectional area
a_i	amplitude
b	beam width
C_t	generalized torsional rigidity
$\widehat{D}$	discriminant
E_{ii}	principal and longitudinal material constants
$\langle EI \rangle_{eq}$	bending rigidity
G_{ii}	directionally dependent shear modulus
$\langle GJ \rangle_{eq}$	torsional rigidity
h	beam thickness
I	second moment of area about the x axis
$\mathbf{I}$	identity matrix
I_α	polar mass moment of inertia per unit length
i	complex number or subscript 1, 2, 3 ..
J	polar moment of inertia
$\langle K \rangle_{eq}$	coupling rigidity
k_i	wavenumber
L	beam length
M	bending moment
N	mode count
$n(\omega)$	modal density
Q	shear force
T	Torque
t_i, s_i	amplitude ratio
t	time
v	transverse displacement
x, y, z	coordinate axes
$\widetilde{\alpha}$	fibre orientation
$\widehat{\phi}$	slope due to bending
κ	shear correction factor
$\langle \kappa AG \rangle_{eq}$	shear rigidity
ν_{ii}	Poisson ratio
ρ	density
ω	frequency
τ_i, θ_i	phase
ψ	torsional rotation

inertia and shear. Mei [5] used a wave approach to determine the natural frequencies and mode shapes of composite beams based on the equations of motion derived in Ref. [4].

Broadband excitation of structures and the resulting noise and vibration is a common engineering problem. The choice of an appropriate technique for analysis depends on the size of the wavelengths that propagate in a structure. In particular; at high frequencies where the wavelength is comparable to or smaller than the dimensions of the structure, it is known that small unavoidable changes in geometry and material properties can on lead to a variance in response over a set of nominally identical systems. Within this range a stochastic technique, Statistical Energy Analysis (SEA), is used to predict vibratory response of structures [6]. A fundamental requirement of SEA is to group the modes of vibration of a structure into specific frequency bands. As a result expressions for the modal density of structures which is defined as the number of modes per unit frequency is compulsory for performing SEA. A compendium of formulas exists for isotropic beams, plates and shells and orthotropic plates [7]. Langley [8] derived a general formula for the modal density of thin anisotropic plates and shells. The work presented herein deals with the modal density of fibre reinforced composite beams having circumferentially asymmetric stiffness or unbalanced layups leading to coupling in bending and torsion.

Recently the authors [9] derived expressions and discussed several important properties of the modal density for fibre reinforced composite Euler-Bernoulli beams where the effects of shear and rotary inertia were neglected. These effects are known to be important when the beams are relatively thick or the wavelengths are small compared to the beam's cross sectional dimensions. In this study the principle of wave train phase closure is the basis for deriving an expression for the modal density of a fibre reinforced composite beam coupled in bending and torsion; including shear deformation and rotary inertia. A graphical approach is used to illustrate the nature of the structural waves supported on the beam. Since the modal density is a parameter usually reserved for high frequency analysis, the influence of shear deformation and rotary inertia is significant. Subsequent to the derivation of an expression for the modal density a number of numerical simulations are conducted to investigate the effects of shear deformation and rotary inertia. The simulations show that the modal density in a frequency band is (1) insensitive to boundary conditions and (2) increases significantly when shear deformation and rotary inertia are included. In each frequency band there exists a fibre orientation that corresponds to a minimum modal density. This particular orientation is shown to not only be dependent on the bending, torsion and coupling rigidities but also dependent on the shear rigidity. Mei [5] showed that the presence of a wave-mode transition for composite beams, coupled in bending and torsion, occurs at a frequency independent of material coupling. The consequence of the wave-mode transition on the modal density is discussed and a separate analytical expression for this special case is given.

2. Modal density of a composite beam

2.1. Equations of motion and the dispersion relation

The modal density of a structure is a parameter that has profound significance and physical interpretation at high frequencies where the spatial wavelength becomes comparable to the cross-sectional dimensions. This consequence is the main motivation for using a beam theory that includes the effects of shear deformation and rotary inertia as the basis for deriving an expression for the modal density of a composite beam. Researchers have also cited the characteristically low shear moduli of fibre reinforced composites as justification for including the shear deformation [10,11]. The equations of motion of a fibre reinforced composite