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Non-reciprocal elastic wave propagation in 2D phononic membranes with spatiotemporally varying material properties

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ABSTRACT

One-dimensional phononic materials with material fields traveling simultaneously in space and time have been shown to break elastodynamic reciprocity resulting in unique wave propagation features. In the present work, a comprehensive mathematical analysis is presented to characterize and fully predict the non-reciprocal wave dispersion in two-dimensional space. The analytical dispersion relations, in the presence of the spatiotemporal material variations, are validated numerically using finite 2D membranes with a prescribed number of cells. Using omnidirectional excitations at the membrane's center, wave propagations are shown to exhibit directional asymmetry that increases drastically in the direction of the material travel and vanishes in the direction perpendicular to it. The topological nature of the predicted dispersion in different propagation directions are evaluated using the computed Chern numbers. Finally, the degree of the 2D non-reciprocity is quantified using a non-reciprocity index (NRI) which confirms the theoretical dispersion predictions as well as the finite simulations. The presented framework can be extended to plate-type structures as well as 3D spatiotemporally modulated phononic crystals.

1. Introduction

Considerable effort has been devoted over the last few decades to study phononic materials (PMs) [1,2]. PMs are engineered structures that are formed by a periodic variation of acoustic or elastic properties (stiffness and inertia) that extend along the dimensions of 1D, 2D or 3D structures. Such periodic variations are typically realized via changes in material property, geometry, or both [3–5]. Periodicity in PMs results in the creation of phononic bandgaps attributed to Bragg scattering effects which can be both theoretically predicted and experimentally observed [6,7]. Phononic bandgaps primarily depend on the unit cell size and are characterized by continuous frequency ranges within which incident wave propagation is significantly impeded in the PM medium. As a result, such materials have been shown to provide promising solutions to challenges in vibration mitigation [8,9], active vibration control [10], wave guidance [11,12], energy harvesting [13], and fluid flow control [14].

Owing to their periodicity, the wave propagation characteristics of large PMs can be reasonably predicted using the band structure of the individual unit cell which portrays the cell's dispersion relations. The dispersion relations relate the wave's frequency ω to its spatial characteristics (e.g. wavenumber k or vector $\mathbf{k}$ for multi-dimensional systems) [15]. Due to elastodynamic reciprocity, band structures of perfectly periodic structures are symmetric about its origin ($\mathbf{k} = \mathbf{0}$) implying that waves travel from point A to B in the same manner they would travel from B to A under similar conditions [16]. Breaking such reciprocity creates a bias in the band structure intended to force waves to travel asymmetrically in opposing directions [17–20]. Non-

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reciprocity in PMs have been very recently utilized to synthesize, among others, acoustic guides [21] and static displacement amplifiers [22]. Means to induce a non-reciprocal behavior include introduction of large nonlinearities, geometrical asymmetries, topological features, and material fields that simultaneously travel in space and time [23,24]. The latter has been recently shown in 1D phononic lattices [25], locally resonant acoustic metamaterials [26], and 1D flexural beams [27,28] to reveal intriguing unidirectional wave propagation capabilities.

The analysis of spatiotemporally modulated structures has, thus far, been limited to 1D structures and is yet to be mathematically implemented in multi-dimensional phononic systems. As such, an adequate investigation of elastic non-reciprocity in phononic 2D elastic structures remains lacking. In this effort, we introduce a comprehensive mathematical framework to capture and predict the non-reciprocal wave dispersion in thin 2D phononic membranes with spatiotemporally modulated (STM) material fields. We begin with the governing motion equations for time- and space-varying material properties that happen simultaneously and independently in both directions within the 2D space. The framework is derived for the general case for any arbitrary periodic modulation waveform and is, in the subsequent sections, mathematically implemented in spatiotemporally modulated membranes with harmonic and square wave modulations of material density and the membrane tension. The theoretically obtained dispersion surfaces are used to show the non-reciprocal behavior in the different propagation regions as well as the corresponding distortion in the irreducible Brillouin zone (IBZ) as a result of the time-traveling material fields. The theoretical dispersion patterns are then verified numerically using actual dispersion contours reconstructed from the displacement response of a finite membrane undergoing the material space and time modulations. Next, a directivity analysis is presented to examine the non-reciprocal behavior of the membrane from the standpoint of the different propagation directions and how the resultant attributes relate to the fundamental direction in which the membrane properties travel. Finally, a non-reciprocity index (NRI) is defined and is used to efficiently summarize the non-reciprocal behavior as a function of the propagating direction and the material modulation speed within the considered frequency ranges.

2. 2D non-reciprocal dispersion analysis

Membranes are ultra-thin 2D structures which sustain transverse loading using in-plane stresses and have no bending (flexural) stiffness. Their vibration dynamics are captured by the 2D wave equation which can be written as

$$\frac{\partial}{\partial x}\left[P(x,y,t)\ \frac{\partial w(x,y,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[P(x,y,t)\ \frac{\partial w(x,y,t)}{\partial y}\right]=\frac{\partial}{\partial t}\left[\rho(x,y,t)\ \frac{\partial w(x,y,t)}{\partial t}\right] \tag{1}$$

where $P(x,y,t)$, $\rho(x,y,t)$, and $w(x,y,t)$, written henceforth as P, ρ, and w for brevity, represent general expressions of the membrane tension, density per unit area, and transverse displacement, respectively, at any given location (x,y) and time instant t. To break time-reversal symmetry and onset non-reciprocal dispersion, we assume the membrane properties P and ρ to follow a traveling-wave like pattern. As a result, these material properties travel simultaneously in space and time, and have no permanent nodes or constant values within the structure. In here, these spatiotemporal modulations of P and ρ are limited to periodic functions that follow the constraints outlined by Cassedy *et. al* [29,30] to avoid an unstable response. In section 3, we will discuss in details two types of modulations: harmonic (sinusoidal) and square wave modulations. In the general case, the periodic spatial modulations have cyclic lengths of $\lambda_{mx}=2\pi/k_{mx}$ and $\lambda_{my}=2\pi/k_{my}$ where k_{mx} and k_{my} are the spatial modulation frequencies in the x and y directions, respectively. The temporal modulations have a periodic time of $T_m=2\pi/\omega_m$ where ω_m represents the temporal modulation frequency. Owing to their periodic nature, P and ρ can be expanded as two successive Fourier series as follows

$$\rho(x,y,t)=\sum_{r=-\infty}^{\infty}\sum_{s=-\infty}^{\infty}\rho_{r,s}e^{ir(\omega_m t-k_{mx}x)}e^{is(\omega_m t-k_{my}y)} \tag{2}$$

and

$$P(x,y,t)=\sum_{r=-\infty}^{\infty}\sum_{s=-\infty}^{\infty}P_{r,s}e^{ir(\omega_m t-k_{mx}x)}e^{is(\omega_m t-k_{my}y)} \tag{3}$$

where $i=\sqrt{-1}$ and the Fourier coefficients, $\rho_{r,s}$ and $P_{r,s}$, can be determined from the triple integrals

$$\rho_{r,s}=\frac{1}{T_m\lambda_{mx}\lambda_{my}}\int_0^{T_m}\int_{-\lambda_{my}/2}^{\lambda_{my}/2}\int_{-\lambda_{mx}/2}^{\lambda_{mx}/2}\rho(x,y,t)e^{-ir(\omega_m t-k_{mx}x)}e^{-is(\omega_m t-k_{my}y)}\,dx\,dy\,dt \tag{4}$$

and

$$P_{r,s}=\frac{1}{T_m\lambda_{mx}\lambda_{my}}\int_0^{T_m}\int_{-\lambda_{my}/2}^{\lambda_{my}/2}\int_{-\lambda_{mx}/2}^{\lambda_{mx}/2}P(x,y,t)e^{-ir(\omega_m t-k_{mx}x)}e^{-is(\omega_m t-k_{my}y)}\,dx\,dy\,dt \tag{5}$$

Using the Floquet theory, traveling waves in a periodic structure comprise the periodicity of the structure which carries them [31,32]. Consequently, a periodic wave solution to Eq. (1) can be written as

$$w(x,y,t)=e^{i(\omega t-\mathbf{k}.\mathbf{r})}\,\overline{w}(x,y,t) \tag{6}$$