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Dynamic response bound analysis for elastic beams under uncertain excitations

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ABSTRACT

By introducing the interval process model, a non-random vibration analysis method is proposed in this paper to calculate the dynamic displacement response bounds of elastic beams under uncertain excitations. Firstly, based on the interval process model, a conception of spatial-time interval field is proposed to describe the uncertainty of excitations in spatial and time domain. Secondly, the middle point function and the radius function of displacement response of the elastic beam are deduced by combining the spatial-time interval field and the traditional mode superposition method, based on which an analytical formulation of the upper and lower displacement response bound functions of the elastic beam are obtained. In addition, the displacement response bound functions of a simply supported beam structure under two types of external excitations, namely the concentrated load and the sinusoidal distributed load, are given. Finally, several numerical examples are investigated to demonstrate the effectiveness of the proposed method.

1. Introduction

In practical engineering, many structures, such as bridges, runways, rails, roadways, pipelines, etc., can be simplified as elastic beams [1]. Therefore, the vibration analysis of elastic beams has been widely studied by many researchers. When the excitations applied to an elastic beam are uncertain or random, the responses of the beam will also exhibit uncertainty, which thus leads to an important random vibration problem. Bogdanoff and Goldberg [2] calculated the mean square displacement and stress of a simply supported Bernoulli-Euler beam subjected to several types of random excitations. Hosseini and Khadem [3] studied the vibration and reliability of a rotating beam with uncertain material properties under random excitations. Zembaty [4] studied the band-limited, non-stationary random vibration of a shear beam. Using the eigenfunction and variation of parameters techniques, Galal et al. [5] obtained a general solution for beams under random loading. Meanwhile, the average and variance of the beam deflection, shear and bending moments were also obtained. Silva et al. [6] obtained approximate solutions of the stochastic beam bending on Winkler foundation by using the Askey-Wiener scheme and the Galerkin method. Kuttler et al. [7]presented a model for vibration analysis of a viscoelastic Gao Beam allowing for stochastic inputs. Shen et al. [8,9] studied the responses of elastic rods, shear beams and Bernoulli–Euler beams with random field properties under random field forcing with linear, Matérn, Cauchy and Dagum covariances. Elishakoff et al. [10]investigated the deformation of deterministic beams and stochastic beams subjected to random excitations, and obtained the exact

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solutions for four kinds of problems. Feng et al. [11] investigated the non-linear integro-differential equations of motion for a slender cantilever beam subjected to axial narrow-band random excitations. Xia and Fujino [12] used numerical methods to study the auto-parametric vibration of a cable-stayed-beam structure under random excitation. Do [13] derived the motion equations of the extensible and slender beams with large motions under both deterministic and stochastic external loads. Brzeźniak et al. [14]studied the equation of motion for an extensible beam under white noise excitations, and proved the existence of global mild solutions and the asymptotic stability of the zero solution by using Lyapunov functions techniques. Pavlović et al. [15]analyzed the stochastic instability problem for an axially loaded Timochenko beam made of viscoelastic material. Anh et al. [16] proposed a version of the regulated stochastic linearization technique for nonlinear random vibrations of Bernoulli-Euler nonlinear beams. Agrawal [17] presented a general analytical technique for stochastic analysis of a continuous beam whose damping characteristic is described using a fractional derivative model. By the way, due to its importance, the uncertainty issue was also widely investigated in the others types of structures [18–22] as well as elastic beams.

Among the above random vibration analyses of elastic beams, the stochastic process model should be firstly established for each random excitation, and hence a great number of time-history testing samples of the excitation are generally required to achieve the precise probability distribution characteristics of the stochastic process. However, in practical engineering problems, because of restrictions in experimental condition or cost, it is often difficult or even impossible to obtain sufficient experimental data. Thus, some assumptions have to be made when using stochastic process model to analyze the random vibration problems of elastic beams. Nevertheless, there are researches indicating that even a small deviation in the probability distribution may result in an extremely large error in structural uncertainty analysis [23].

This paper therefore proposes a non-random vibration analysis method for elastic beams based on our newly developed interval process model [24,25], which could efficiently calculate the dynamic response bounds of an elastic beam under uncertain excitations. In the interval process model, an interval rather than a precise probability distribution is used to describe the uncertainty of a time-varying parameter at each time point, and two boundary curves are then employed to depict the whole time-varying uncertainty of the parameter. The interval process model thus provides a very simple mathematical tool for time-varying uncertainty modeling of problems without sufficient experimental data. In this paper, based on the interval process model, a conception of spatial-time interval field is firstly given to describe the uncertainty of excitations applied to the elastic beam in spatial and time domain. Then in the vibration analysis of the beam, the excitation and response are both given in the form of spatial-time interval field, which avoids the introduction of probability characteristics. In practical engineering the obtained response bounds will be very helpful for reliability evaluation and safety design of an elastic beam. Additionally, the conception of response bounds is generally very easy to understand and use for engineers in practical structural design. The remainder of this paper is organized as follows: Section 2 introduces the fundamentals of interval process model; Section 3 presents the definition of spatial-time interval field; Section 4 gives the formulation of the proposed non-random vibration analysis method for elastic beams; Section 5 gives the formulation of dynamic displacement response bounds for simply supported beams under two different types of loads; Section 6 provides some numerical examples; Section 7 finally summarizes the conclusions of this paper.

2. Fundamentals of interval process model

As shown in Fig. 1, the interval process model proposed by the authors [24–26] employs a bounded and closed interval for quantification of the parametric uncertainty at arbitrary time point, and defines a specific auto-correlation coefficient function for the description of correlation between the interval variables at arbitrary two different time points.

Definition 1. A time-varying uncertain parameter $\{X(t), t \in T\}$ is an interval process if for arbitrary time $t_i \in T, \quad i = 1, 2, \ldots$, the possible values of $X(t_i)$ can be represented by an interval $X^I(t_i) = [X^L(t_i), X^U(t_i)]$, where T is a parameter set of t.

Definition 2. For an interval process $X^I(t)$ with the upper bound function $X^U(t)$ and the lower bound function $X^L(t)$, the middle point function of $X^I(t)$ is defined as:

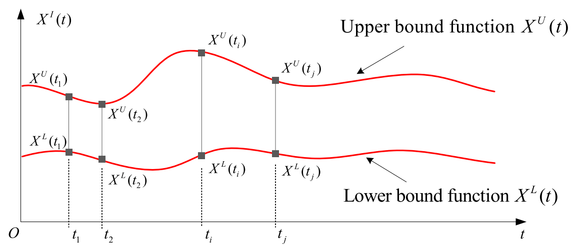

Fig. 1. The interval process [25].