Journal of Sound and Vibration 422 (2018) 79–91

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Impact of excitation waveform on the frequency stability of electrostatically-actuated micro-electromechanical oscillators

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A R T I C L E I N F O

Article history:
Received 25 August 2017
Received in revised form 3 January 2018
Accepted 13 February 2018
Available online 22 February 2018

Keywords:
MEMS sensors
Oscillators
Nonlinear actuation
Frequency stability
Phase noise

A B S T R A C T

In this paper, the frequency stability of high-Q electrostatically-actuated MEMS oscillators with cubic restoring forces, and its relation with the amplitude, the phase and the shape of the excitation waveform, is studied. The influence on close-to-the carrier frequency noise of additive processes (such as thermomechanical noise) or parametric processes (bias voltage fluctuations, feedback phase fluctuations, feedback level fluctuations) is taken into account. It is shown that the optimal operating conditions of electrostatically-actuated MEMS oscillators are highly waveform-dependent, a factor that is largely overlooked in the existing literature. This simulation-based study covers the cases of harmonic and pulsed excitation of a parallel-plate capacitive MEMS resonator.

1. Introduction

The ultimate performance of MEMS resonant sensors is limited by frequency fluctuations, typically caused by additive noise processes such as thermomechanical noise, as well as long-term effects, such as near-DC flicker noise, modulated close to the carrier through parametric effects [1]. Both sorts of phenomena are studied in the literature [2,3] under the assumption that the resonator can be described as a Duffing resonator, with a cubic restoring force.

Although the Duffing model may be accurate in some cases, it fails to capture the behavior of electrostatically-actuated MEMS resonators. A good example of the specificity of the capacitive nonlinearity lies in the existence of “catastrophic” static and dynamic instabilities [4,5] which set the ultimate limits of operation of electrostatic MEMS resonators. Recently, we have shown that the resonant pull-in phenomenon is highly dependent on the excitation waveform used to maintain the resonator in an oscillation state [6,7]. In fact, it is shown in Ref. [7] that the maximal stable oscillation amplitude of electrostatically-actuated parallel-plate resonators with high quality factors (Q) and small bias voltage (with respect to the static pull-in voltage) is strictly waveform-dependent, i.e. it depends on nothing else than the amplitude, the phase and the shape of the excitation waveform.

Given the impact of the excitation waveform on the pull-in instability, it stands to reason to suspect it also has an impact on the frequency stability of electrostatically-actuated resonant sensors: in Ref. [8], we showed that this was indeed the case and derived frequency stability optima for harmonically-excited and pulse-excited resonators. However, the study conducted in Ref. [8] is limited to an ideal framework in which only the electrostatic nonlinearity is considered and the other

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https://doi.org/10.1016/j.jsv.2018.02.030

nonlinearities are neglected. The motivation of the present work is to extend the results of [8] to the more realistic case when the Duffing nonlinearity –which typically results from axial stress buildup in flexural resonators, i.e. "stress-stiffening"– is also accounted for. This is an important addition to our previous work because the Duffing nonlinearity is ubiquitous in MEMS resonators.

Several recent papers [9–14] have proposed to improve frequency stability or the overall linearity of the frequency response of MEMS resonators in various ways. For example, in Ref. [14], the influence of the Duffing nonlinearity is decreased (but not cancelled) by optimizing the mechanical design of the resonator. Another typical scheme [9–12] is to cancel out the "hardening" Duffing nonlinearity with a "softening" nonlinearity by adjusting the bias voltage of the resonator. This approach can be successful to minimize the impact of the "A-f effect", i.e. the conversion of amplitude noise to frequency noise in nonlinear oscillation regimes. Most of these efforts are based on a 3^{rd}- or 5^{th}-order Taylor series approximation of the electrostatic force, so that they are limited to moderate oscillation amplitudes (with respect to the electrostatic gap). Furthermore, they neither study the impact of the excitation waveform on stability or linearity, nor that of parametric fluctuations (e.g. fluctuations of the bias voltage, of the excitation voltage, etc.). On the other hand, a rather complete study of the stability of MEMS oscillators, with or without parametric fluctuations, is made in Refs. [3] and [13]. However the framework of this study is limited to Duffing resonators with harmonic excitation and a linear actuation scheme, and its results cannot be applied to electrostatic resonators.

The present paper is aimed at filling the gaps between the above-mentioned studies. It is based on a qualitative, simulation-based investigation of frequency stability in high-Q electrostatically-actuated MEMS oscillators with cubic restoring forces. Instead of focusing on a single aspect of the problem, we systematically study the influence of the amplitude, the phase and the shape of the excitation waveform on close-to-the carrier frequency noise, induced by additive (e.g. thermomechanical noise) or parametric processes (bias voltage fluctuations, feedback phase fluctuations, excitation fluctuations). This study is led for different ratios of the nonlinear hardening/softening parameters, in order to observe nonlinearity cancellation effects as in Refs. [9–13], but it is not based on polynomial approximations of the electrostatic force, so that its results are valid regardless of the mechanical oscillation amplitude. The impact of the excitation waveform on our results is illustrated through three examples, with different characteristics, shown in Fig. 1: a harmonic waveform and two waveforms based on short voltage impulses. The last two cases illustrate how frequency stability is influenced by the localization in time of the energy supplied by the feedback loop to the resonator, and also by the skewness of the excitation waveform.

In Section 2, we present the theoretical framework of our analysis and the steady-state model of the oscillator. In Section 3, the general approach for assessing frequency stability based on a sensitivity analysis is presented. A simplified analysis is then proposed, in order to derive tractable results and gain insight into the different phenomena, and applied to additive perturbations (Section 4) and to parametric fluctuations (Section 5). Simulation-based illustrations of our results are given and commented in Section 6. Finally, Section 7 contains some concluding remarks and perspectives for future work.

2. Steady-state solution of SDOF-model

In this section, we develop the model of a single-degree of freedom (SDOF) one-sided parallel-plate resonator actuated with voltage $V(t) = V_{exc}(t) + V_b$, where V_b is a constant bias voltage, and $V_{exc}(t)$ the excitation voltage, corresponding to the idealized geometry of Fig. 2. Provided the lateral dimensions of the resonator are large with respect to the gap between the electrodes, fringing fields may be neglected, and the electrostatic forces are governed by the plane capacitance approximation [15]. Then, assuming that $v(t) \ll 1$, where $v(t) = V_{exc}(t)/V_b$, the motion of the resonator is accurately described by:

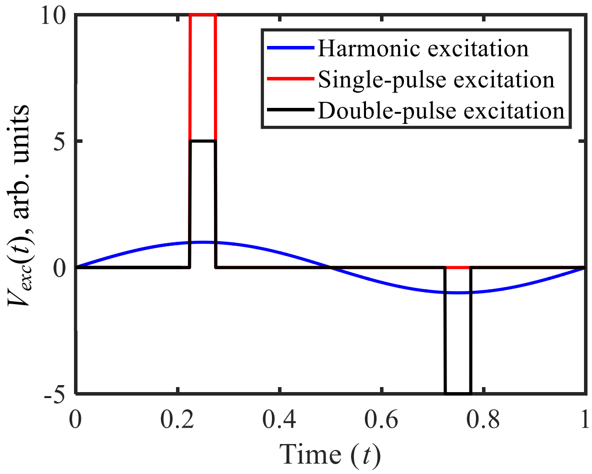

Fig. 1. Excitation waveforms considered in this work. The amplitudes are chosen so that all waveforms have the same first harmonic power.