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Band-gap analysis of a novel lattice with a hierarchical periodicity using the spectral element method

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ABSTRACT

Inspired by the hierarchical structures of butterfly wing surfaces, a new kind of lattice structures with a two-order hierarchical periodicity is proposed and designed, and the band-gap properties are investigated by the spectral element method (SEM). The equations of motion of the whole structure are established considering the macro and micro periodicities of the system. The efficiency of the SEM is exploited in the modeling process and validated by comparing the results with that of the finite element method (FEM). Based on the highly accurate results in the frequency domain, the dynamic behaviors of the proposed two-order hierarchical structures are analyzed. An original and interesting finding is the existence of the distinct macro and micro stop-bands in the given frequency domain. The mechanisms for these two types of band-gaps are also explored. Finally, the relations between the hierarchical periodicities and the different types of the stop-bands are investigated by analyzing the parametrical influences.

1. Introduction

Elastic wave propagation in periodic structures has been attracting a wide attention for many years. Elastic waves in the pass-band frequency range can propagate well through the periodic structures, while those in the stop-band frequency range can hardly propagate [1–4]. This characteristic is called the band-gap property of periodic structures (or phononic crystals). The band-gap mechanisms of the phononic crystals can be classified into the Bragg scattering mechanism [1,2] and the local resonance mechanism [3,4]. According to the Bragg scattering mechanism, the band-gaps result from the scattered elastic waves generated by the periodically arranged scatterers. Those based on the local resonance mechanism are generated due to the resonance of the local vibrators. Due to the band-gap property, periodic structures can be utilized as mechanical filters for wave propagation. Mead [5,6] has made substantial contributions to the analysis and prediction of elastic wave propagation in periodic engineering structures. Since then, considerable investigations on various types of periodic structures have been conducted, such as spring-mass systems [7], periodic beams [8], periodic plates [9,10], lattices [11–13], sandwich structures [14] and so on.

In nature, various creatures have evolved into peculiar hierarchical structures to adapt to the environment over millions of years of natural selection. Many animals or plants possess distinctive body surfaces which are hydrophobic, anti-corrosive,

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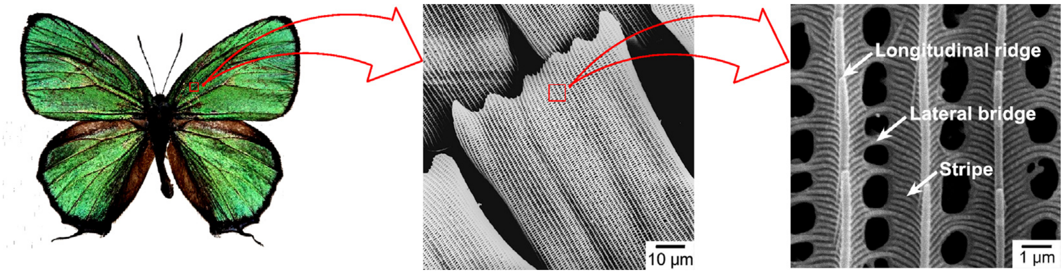

Fig. 1. Multi-dimensional view of the butterfly wing surfaces [15].

anti-fouling, anti-icing, drag reducing, self-cleaning, anti-wearing and so on [15–19]. Fig. 1 shows a multi-dimensional view of the butterfly wing surfaces. It is interesting to note that the butterfly wing possesses macro and micro periodicities which benefit weight reduction and hydrophobicity. Inspired by the butterfly wing in different dimensions, one can conjecture that structures with a hierarchical periodicity may display distinctive functions in terms of the elastic wave propagation. Based on the Bragg scattering mechanism, the hierarchical periodicity may enhance the wave scattering between the unit-cells, which should be beneficial to the Bragg band-gaps. The present work seeks to systematically investigate the lattice structures with a two-order hierarchical periodicity and it aims at providing a basis for further researches on the structures with a multi-order hierarchical periodicity.

Considerable efforts have already been devoted to the wave propagation in periodic structures by numerous methods. Liu et al. theoretically and numerically studied the behaviors of the in-plane and out-of-plane wave propagation in a curved phononic crystal beam using the transfer matrix method in combination with the Bloch theorem [8]. Zhang et al. investigated the tunability of the in-plane wave propagation in two-dimensional Terfenol-D/epoxy phononic crystal plates considering the magneto-mechanical coupling of the magnetostrictive materials by means of the plane wave expansion method [20]. Kulpe et al. computed the external acoustic scattering by phononic crystals with an arbitrary exterior shape using a Bloch wave expansion technique coupled with the Helmholtz-Kirchhoff integral [21]. Carta et al. analyzed the dynamic flexural behavior of the "Brabau" bridge across the river named "Tirso" in Italy using the finite element method (FEM) [22].

Considering the band-gap property analysis of lattices with a hierarchical periodicity, the significance of the establishment of an efficient dynamic model is evident. Two issues should be taken into account in this regard. Firstly, periodic structures are generally very complex because they contain a large amount of sub-structures. Secondly, highly accurate solutions in the frequency domain are required while analyzing the dynamic behaviors of the systems. Although the rather complex periodic structures can be solved by the FEM, the computational cost will increase substantially as the size or the number of the sub-structures increases. Moreover, the solution accuracy will deteriorate at high frequencies. Hence, there is an indispensable need to develop a suitable and efficient method to study the dynamic behaviors of the lattices with a hierarchical periodicity.

Recently, the spectral element method (SEM) has been extended by the authors to solve the dynamic behaviors of various types of complex periodic structures [9–14]. The SEM combines the key features of the FEM, the dynamic stiffness method and the spectral analysis method, which makes this approach possible to mesh and assemble a set of finite elements with a minimum number of degrees of freedom, and to superpose the wave modes via the Fourier transform technique. Two significant advantages have been demonstrated in the model establishment, i.e., the reduced element number and the high accuracy of the results in the frequency domain [23,24]. This approach is based on a Fourier-transform analysis. Since a geometrically and materially uniform structure can be considered as a single spectral element, the number of the elements can be largely reduced. Furthermore, the method is directly derived from the wave equation and the mass is distributed accurately across the element, which ensures the high accuracy.

The aim of this study is to propose and design a new type of lattices with a two-order hierarchical periodicity. The SEM is explored to establish the equations of motion of the whole system. The band-gap properties are analyzed and discussed based on the highly accurate results. An interesting phenomenon, i.e. the existence of distinct macro and micro stop-bands, is originally observed. The relations between the hierarchical periodicities and the characteristics of the stop-bands are investigated in details.

2. Problem description

In this section, a periodic lattice with n unit-cells in the global coordinate system ($x^g - y^g$) as shown in Fig. 2(a) is investigated. The unit-cell is constructed by four beams with a length L and it is displayed in Fig. 2(b). The shape of the unit-cells can be adjusted by the angle θ. The beam in the unit-cells with a circular cross-section can be periodic as shown in Fig. 2(c) or uniform as displayed in Fig. 2(d). The number of the sub-unit-cell in a periodic beam as shown in Fig. 2(c) is n_0, and the length of the sub-unit-cell is $a_0 = L/n_0$. The length of the material M_1 in the sub-unit-cell is a_1, and the ratio of the lengths of the material M_1 and the sub-unit-cell is $\zeta = a_1/a_0$. When a periodic beam is applied in the periodic lattice, the whole