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A hierarchical Bayesian method for vibration-based time domain force reconstruction problems

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ABSTRACT

Traditional force reconstruction techniques require prior knowledge on the force nature to determine the regularization term. When such information is unavailable, the inappropriate term is easily chosen and the reconstruction result becomes unsatisfactory. In this paper, we propose a novel method to automatically determine the appropriate q as in ℓ_q regularization and reconstruct the force history. The method incorporates all to-be-determined variables such as the force history, precision parameters and q into a hierarchical Bayesian formulation. The posterior distributions of variables are evaluated by a Metropolis-within-Gibbs sampler. The point estimates of variables and their uncertainties are given. Simulations of a cantilever beam and a space truss under various loading conditions validate the proposed method in providing adaptive determination of q and better reconstruction performance than existing Bayesian methods.

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1. Introduction

In engineering practice, the knowledge of input force is critical to the design and analysis of structures. However, the acquirement of such knowledge is difficult. There is often no appropriate positions to mount force sensors, and sometimes mounting such sensors will greatly weaken the stiffness of structures. Instead of directly measuring forces, we turn to the inverse approach. We will rely on knowledge about the structure and measurements of its responses to inversely calculate the forces applied on it.

Unfortunately, the inverse approach is usually mathematically ill-posed. It means that an inevitable small amount of measurement noise would cause great variation to the inversely estimated force, thus making it unreliable. This ill-posedness is due to the fact that the singular values of system matrix have lots of small values (ill-conditioned case), which plays an important role in amplifying the effect of the measurement noise during inverse calculation. Sometimes there is a noticeable drop between the singular values (rank-deficient case), making the direct inversion not operable [1]. So most of force reconstruction studies focus on the development and application of techniques to ease such ill-posedness. Liu et al. [2] applied moving least squares fitting to approximate the unknown force with a few smooth functions in time-concentrated support domains. The condition number of the transformed system matrix is effectively improved thanks to the smooth basis functions. Li et al. [3] applied multi-resolution wavelet bases to decompose the unknown force. The decomposition level is carefully handled to balance the well-condition of the reformed system matrix and the loss of reconstruction resolution. These two methods share the same idea in dealing with the ill-posedness of force reconstruction problems: apply reasonable transformation to the unknown force

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history and well-pose the reformed system matrix. Similar ideas can be found in Refs. [4–7].

Another widely adopted technique is regularization. Instead of directly manipulating the system matrix, regularization techniques stabilize the solution by adding a penalty on highly oscillating components. Tikhonov regularization, probably the most well-known one, considers the ℓ_2-norm of the force as a regularization term. The amount of regularization is controlled by a parameter and determined by a certain criterion, such as L-curve criterion. Jacquelin et al. [1] compared the performance of Tikhonov regularization and Truncated Singular Value Decomposition (TSVD) on impact identification of circular plates. The difference between two widely used parameter determination criterions, L-curve and Generalized Cross Validation (GCV) is also given. Liu and Shepard [8] performed force reconstruction in frequency domain. The singular values of Frequency Response Function (FRF) matrix are checked at each frequency for the necessity of regularization. The method saved as much original information as possible and avoided ill-posed inversion at the same time. The force history is computed by inverse Fourier transform of the estimated force spectrum. Other methods considering Tikhonov regularization can be found in Refs. [9–12].

Recently, the sparsity-inducing property of ℓ_1-norm regularization has captured much attention. Contrary to Tikhonov or ℓ_2-norm regularization, which is more suitable for continuous-type solutions, ℓ_1-norm regularization favours sparse-type solutions such as impacts, especially in low Signal-to-Noise Ratio (SNR) conditions [13]. It is worth mentioning that for time-domain and frequency-domain approaches, sparse-type and continuous-type solutions usually stand for different things. For time-domain approaches, the regularization term is imposed on the force history. ℓ_1-norm regularization in this case favours sparse-profile forces such as impacts; while for frequency-domain source reconstruction approaches like [14], the regularization term is imposed on the spatial force vector at each frequency. ℓ_1-norm regularization in this case favours spatially sparse forces such as concentrated forces. Samagassi et al. [15] adopted ℓ_1-norm regularization on multiple impact reconstruction on a cantilever beam and Relevance Vector Machine (RVM) for the solution. Qiao et al. [16] applied sparse reconstruction by separable approximation (SpaRSA) in Ref. [17] to reconstruct impact forces based different bases/dictionaries. Qiao et al. [18] applied the primal-dual interior point method (PDIPM) to solve a large-scale impact force reconstruction problem. Rezayat et al. [19] introduced grouping technique into force identification problems. They applied a $\ell_{2,1}$-mixed-norm regularization term, favouring continuity within groups and sparsity between groups, so that the algorithm is able to localize a single force from a few possible locations regardless of the force spectrum. This approach have been extended by Aucejo and De Smet [20] to allow more flexibility by using $\ell_{2,q}$-mixed-norm regularization. In Ref. [21], a compromised global $\ell_{1.1}$-norm was used to reconstruct a point force on the surface of a plate and its boundary forces. In Ref. [14], the structure is divided into several identification regions that allows setting the value of q to a reasonable value more easily. $\ell_{0.5}$-norm and ℓ_2-norm were used for the point force and boundary forces separately, and better results were achieved. However, careful analysis of the mechanical problem is still needed to infer appropriate values of q.

In engineering practice, prior information on the sparsity/continuity of the unknown force is usually lacked. In such cases, the value of q should also be inferred as part of the reconstruction problem. In this paper, a novel hierarchical Bayesian approach is proposed for time-domain force history reconstruction problems. Bayesian approaches take into account the uncertainties of different variables [22] and preserve an inherent regularization property [23]. Different from Ref. [23], in this contribution the value of q is considered a to-be-determined random parameter in a hierarchical structure and inferred based on measurements of structural vibrations only. A Metropolis-within-Gibbs sampler is applied to draw samples from the posterior distribution of variables. With the sampling procedure, the posterior distributions of random variables are widely explored and uncertainties of their estimates could be evaluated. Finally we need to point out that, recently a similar hierarchical Bayesian formulation is proposed in Ref. [24]. However [24], considers the spatial reconstruction of vibration sources, while this contribution is more focused on the optimization of force history reconstruction. The applied Markov Chain Monte Carlo (MCMC) samplers are also different.

The rest of the paper is organized as follows. Section 2 introduces the general state space model and the construction of the system matrix. Section 3 proposes the hierarchical Bayesian formulation. The relationship between the proposed formulation and traditional deterministic methods is discussed. Section 4 introduces the Metropolis-within-Gibbs sampler. The post-evaluation metrics are also given. Section 5 illustrates the proposed method on numerical simulations of a cantilever beam and a space truss. Comparisons with existing methods are made. The conclusions are drawn in Section 6.

2. Problem formulation

The structure considered in this research is supposed to be linear elastic, and a single concentrated force is acting at a known location. Suppose the structure is spatially discretized with finite element (FE) method. The system governing equation, in a general form, is

$$\mathbf{M}\ddot{\mathbf{y}}(t) + \mathbf{C}\dot{\mathbf{y}}(t) + \mathbf{K}\mathbf{y}(t) = \mathbf{L}f(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K} \in \mathbb{R}^{n_d \times n_d}$ are the system mass, damping and stiffness matrices; n_d is the number of physical degree of freedoms (DoFs); $\ddot{\mathbf{y}}(t)$, $\dot{\mathbf{y}}(t)$ and $\mathbf{y}(t)$ are respectively acceleration, velocity and displacement vectors at time t. $f(t)$ is the external force at time t. $\mathbf{L} \in \mathbb{R}^{n_d \times 1}$ is the mapping vector for the external force.

The state space representation of Eq. (1) is

$$\dot{\mathbf{z}}(t) = \mathbf{A}_c\mathbf{z}(t) + \mathbf{B}_c f(t) \tag{2}$$

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