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On propagation of axisymmetric waves in pressurized functionally graded elastomeric hollow cylinders

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A B S T R A C T

Soft materials can be designed with a functionally graded (FG) property for specific applications. Such material inhomogeneity can also be found in many soft biological tissues whose functionality is only partly understood to date. In this paper, we analyze the axisymmetric guided wave propagation in a pressurized FG elastomeric hollow cylinder. The cylinder is subjected to a combined action of axial pre-stretch and pressure difference applied to the inner and outer cylindrical surfaces. We consider both torsional waves and longitudinal waves propagating in the FG cylinder made of incompressible isotropic elastomer, which is characterized by the Mooney-Rivlin strain energy function but with the material parameters varying with the radial coordinate in an affine way. The pressure difference generates an inhomogeneous deformation field in the FG cylinder, which dramatically complicates the superimposed wave problem described by the small-on-large theory. A particularly efficient approach is hence employed which combines the state-space formalism for the incremental wave motion with the approximate laminate or multi-layer technique. Dispersion relations for the two types of axisymmetric guided waves are then derived analytically. The accuracy and convergence of the proposed approach is validated numerically. The effects of the pressure difference, material gradient, and axial pre-stretch on both the torsional and the longitudinal wave propagation characteristics are discussed in detail through numerical examples. It is found that the frequency of axisymmetric waves depends nonlinearly on the pressure difference and the material gradient, and an increase in the material gradient enhances the capability of the pressure difference to adjust the wave behavior in the FG cylinder. This work provides a theoretical guidance for characterizing FG soft materials by in-situ ultrasonic nondestructive evaluation and for designing tunable waveguides via material tailoring along with an adjustment of the pre-stretch and pressure difference.

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Nomenclature glossary

B_r, B_0, B_t Undeformed, initial (deformed) and current configurations
$\mathbf{x} = \boldsymbol{\chi}(\mathbf{X}), t$ Finite motion and time
$\mathbf{X}, \mathbf{x}$ Position vectors in B_r and B_t
$\mathbf{N}, \mathbf{n}, \mathbf{n}_t$ Outward unit normal vectors in B_r, B_0 and B_t
$\boldsymbol{\sigma}, \mathbf{T}$ Cauchy and nominal stress tensors
$\mathbf{t}^a, \mathbf{t}^A$ Applied mechanical traction vectors per unit area of ∂B_t and ∂B_r
$W(\mathbf{F})$ Strain energy function per unit reference volume
W_i Derivative of W with respect to principal stretch λ_i
$\mathbf{u} = \dot{\mathbf{x}}, u_i$ Incremental displacement vector and its components
$\dot{q}$ Incremental Lagrange multiplier
$\boldsymbol{\Gamma}, \Gamma_{\alpha i \beta j}$ Referential elasticity tensor and its components
$\dot{\mathbf{t}}^A, \dot{\mathbf{t}}_0^A$ Lagrangian and Eulerian incremental traction vectors on ∂B_r and ∂B_0
A, B, L, a, b, l Inner and outer radii, and length of undeformed and deformed FG-EHC
λ, λ_z Circumferential and axial stretches
$\eta, \overline{\eta}$ Ratios of outer radius to inner radius of undeformed and deformed FG-EHC
Λ, ξ Dimensionless radial coordinates of undeformed and deformed FG-EHC
$\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}$ Initial principal stress components in B_0
N Resultant axial force on each end of deformed FG-EHC
μ_{10}, μ_{20} Material constants in affine variations
G Axisymmetric deformation parameter defined as $G = a^2 - \lambda_z^{-1} A^2$
S Integration constant defined in Eq. (25)
$\mathbf{Y}_1, \mathbf{M}_1, \mathbf{V}_1, \overline{\mathbf{M}}_1$ Incremental state vectors and 2×2 system matrices for T waves
$f_i (i = 1 - 8)$ Material parameters appearing in system matrix $\mathbf{M}$
p_0 Pressure-like quantity
$\overline{K}, \overline{\Omega}$ Dimensionless wave number and circular frequency depending on deformations
c, v_p Phase velocity of incremental waves and its dimensionless counterpart
$\mathbf{S}_k (k = 1, 2)$ Global transfer matrices for T and L waves
$\xi_{j0}, \xi_{j1}, \xi_{jm}$ Dimensionless radial coordinate at inner/outer/middle surfaces of the jth layer
$\mathbf{D}_k (k = 1, 2)$ Coefficient matrices for T and L waves in Eq. (50)
Δp^* Dimensionless pressure difference
$(\Delta p_1^*, \Delta p_2^*)$ Allowable range of the pressure difference corresponding to $\beta = -0.5$
U_r^*, U_θ^*, U_z^* Normalized displacement amplitudes of incremental axisymmetric waves
m Circumferential wave number of non-axisymmetric waves (integer)
$\partial B_r, \partial B_0, \partial B_t$ Boundaries of B_r, B_0 and B_t
$\mathrm{d}A, \mathrm{d}a, \mathrm{d}a_t$ Surface elements in B_r, B_0 and B_t
$\mathbf{b}, \mathbf{C}$ Left and right Cauchy-Green strain tensors
$\mathbf{F}, J$ Deformation gradient tensor and its determinant
ρ_r, ρ Mass density in B_r and B_t (or B_0)
q Lagrange multiplier
$\mathbf{I}, \lambda_i (i = 1, 2, 3)$ Identity tensor and principal stretches
$\sigma_{ii} (i = 1, 2, 3)$ Principal Cauchy stresses
$\dot{\mathbf{T}}_0, \dot{T}_{0ij}$ Push-forward version of Lagrangian increment $\dot{\mathbf{T}}$ and its components
$\mathbf{H}$ Incremental displacement gradient tensor with respect to B_0
$\boldsymbol{\Gamma}_0, \Gamma_{0piqj}$ Instantaneous elasticity tensor and its components
p_a Applied pressure on the boundary ∂B_0
$R, \Theta, Z, r, \theta, z$ Cylindrical coordinates in undeformed and deformed configurations
λ_a, λ_b Circumferential stretches of inner and outer surfaces
H, h Thicknesses of undeformed and deformed FG-EHC
$W^*(\lambda, \lambda_z)$ Reduced strain energy density function for axisymmetric deformations
$p_{in}, p_{ou}, \Delta p$ Pressures on the inner and outer surfaces of FG-EHC in B_0 and pressure difference
$\mu_1(R), \mu_2(R)$ Material parameters depending on the radial coordinate R
β_1, β_2 Material gradient parameters in affine variations
$\mathbf{Y}, \mathbf{M}$ Incremental state vector and 6×6 system matrix
$\mathbf{M}_{ij}$ Four partitioned 3×3 sub-matrices of $\mathbf{M}$
$\mathbf{Y}_2, \mathbf{M}_2, \mathbf{V}_2, \overline{\mathbf{M}}_2$ Incremental state vectors and 4×4 system matrices for L waves