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An analytically-based method for predicting the noise generated by the interaction between turbulence and a serrated leading edge

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ABSTRACT

This paper considers the interaction of turbulence with a serrated leading edge. We investigate the noise produced by an aerofoil moving through a turbulent perturbation to uniform flow by considering the scattered pressure from the leading edge. We model the aerofoil as an infinite half plane with a leading edge serration, and develop an analytical model using a Green's function based upon the work of Howe. This allows us to consider both deterministic eddies and synthetic turbulence interacting with the leading edge. We show that it is possible to reduce the noise by using a serrated leading edge compared with a straight edge, but the optimal noise-reducing choice of serration is hard to predict due to the complex interaction. We also consider the effect of angle of attack, and find that in general the serrations are less effective at higher angles of attack.

1. Introduction

Demand for air travel is booming, and is leading to the expansion of airports and the creation of new routes, with the number of aircraft growing by around 4% a year. Flightpath 2050 [6] has set out a roadmap to the middle of the century with key environmental targets such as a 90% cut in nitrous oxide emissions, a 75% cut in carbon dioxide emissions and a 65% reduction in effective perceived noise, all in comparison with the 2000 levels. The largest contributor at present to the noise is the aircraft engine, although radical future designs may completely change the dominant noise sources.

A modern turbofan has many sources of noise, and one of the key sources is fan noise. Peake and Parry [24] identified several different components such as rotor (or fan) self-noise, rotor-stator (or fan-vane) interaction, rotor casing boundary-layer interaction and droop-fan interaction.

In this paper we will focus on the rotor-stator interaction noise, which has both broadband and tonal elements, and is one of the dominant broadband and tonal sources of noise. The stators (or outlet guide vanes) straighten the swirling flow, but they do so at the expense of creating noise. The rotor wake is the sum of a uniform rotating flow plus a turbulent wake from each rotor blade, with a typical wake evolution shown in Cooper and Peake [[5], Fig. 2]. The turbulent wake from the rotor then interacts with the stators that block the wake, producing broadband noise. There is also tonal noise produced at harmonics of the blade passing frequency. By tuning the turbofan with different combinations of rotor and stator blades we the tonal noise

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can be controlled [25,28].

This study considers flow-blade interaction in the context of the turbulent wake of the rotor hitting a single aerofoil, a stator blade. Several analytical models have been developed for the interaction of turbulence with an aerofoil, such as the theories of Amiet [1] and Howe [11]. Both theories involve considering only the trailing edge noise from a semi-infinite aerofoil and using a Green's function of the half plane. Howe's model assumes that the flow is at a low Mach number and the turbulence is frozen. Amiet's model is valid for all subsonic mean flows, and differs from Howe's in how the turbulence interacts with the aerofoil. Howe extended his theory to both sinusoidal [12] and sawtooth [13] serrations, and showed that serrations reduce the noise. According to his theory, reducing the wavelength of the serrations (and hence making them less shallow) reduces the noise. However, his Green's function was only valid for shallow serrations. Howe also extended his theory to aerofoils with a finite chord [16]. Roger and Moreau [26] extended Amiet's model in two ways. Firstly, they considered three dimensions and secondly they included the effect of back-scattering. Recently, Lyu et al. [20] proposed a new method to consider trailing edge serrations, by generalising Amiet's model to sawtooth serrations.

A very recent paper by Lyu and Azarpeyvand [19] extended Amiet's leading edge noise-prediction model to leading edge sawtooth serrations. Their analytical model compared favourably to experiments, and the noise reduction techniques were examined, with the primary mechanism destructive interference. Using the extension of Amiet's model as in their paper also allows the directivity of the leading edge noise to be closely examined, while Howe's model does not consider the directivity of the noise.

There have been a number of recent experimental and numerical studies on the effect of the serrations. A study from Haeri et al. [10] showed numerically that leading edge serrations reduce aerofoil noise. Additionally, experimental work by Gruber [8], Gruber et al. [9] and Narayanan et al. [23] further validated the theory that leading edge serrations can reduce aerofoil noise by a significant amount. In particular, Gruber [8] showed that Howe's model over-predicts the sound reduction from serrations, due to the assumption of frozen turbulence. However, the Green's function from Howe's method is only valid for shallow serrations, which is not the case for the serrations in Gruber [[8], Fig. 4.4].

Recent studies by Chaitanya et al. [3,4] have considered a variety of different leading edge serrations, such as double-wavelength serrations, and their effect on noise performance and aerodynamic performance experimentally. They showed that it is generally possible to reduce the noise without comprising too much on aerodynamic performance.

In this paper we use the Green's function from Howe's analytical model and the model of turbulence from Haeri et al. [10] to investigate the effect of turbulent flow interacting with an aerofoil with a sinusoidal serrated leading edge. We show it is possible to reduce the noise by using a serrated leading edge, but it is hard to predict the correct choice of serration to minimise the noise.

1.1. Organisation

This paper is laid out as follows. In Section 2 we review Howe's method for calculating the pressure from an aerofoil in unsteady flow, and in Section 3 we derive the necessary Green's function for a straight-edged aerofoil and a serrated aerofoil. In Section 4 we calculate the pressure from the synthetic turbulence in Haeri et al. [10]. In Section 5 we consider synthetic turbulence comprising of a single eddy, and consider the effect of the eddy and flow parameters on the noise. In Section 6 we consider synthetic turbulence comprising of multiple eddies, and again show that the amount of noise reduction and optimal geometry of serrations vary significantly with the properties of the turbulence. Finally, in Section 7 we consider an aerofoil at a small angle of attack, and show that the angle of attack generally reduces the effectiveness of the serrations.

In this paper, we consider for the first time synthetic turbulence generated by eddies in Howe's model. We calculate the scattered pressure analytically, although we need to calculate one integral numerically. It is also the first time that multiple eddies interacting with each other in a non-linear way have been studied analytically in this context.

1.2. Geometry of the aerofoil

We model a single aerofoil blade as an infinitely thin half plane, and introduce a serration function $\mathscr{F}(z)$ (such as a sinusoidal wave or sawtooth) on the leading edge of the aerofoil (Fig. 1). Mathematically, the aerofoil is defined by

$$\{(x,y,z) \in \mathbb{R}^3 | z \in \mathbb{R}, \quad \mathscr{F}(z)\cos\alpha < x < \infty, \quad y = x\tan\alpha\}, \tag{1}$$

where α is the angle of attack of the aerofoil. We introduce serrated cylindrical coordinates of $(r^\star, \theta^\star, z^\star)$, defined by

$$(x,y,z) = (\mathscr{F}(z^\star)\cos\alpha - r^\star\cos\theta^\star, -\mathscr{F}(z^\star)\sin\alpha - r^\star\sin\theta^\star, z^\star). \tag{2}$$

In these modified cylindrical coordinates $r^\star$ is the distance to the leading edge of the aerofoil, $\theta^\star$ is the angle in the x-y plane, and $z^\star$ the height (Fig. 1). Even at zero angle of attack, these coordinates are non-orthogonal, which will present difficulties in calculating the Laplacian later. We define $\theta^\star$ such that the two sides of the aerofoil correspond to $\theta^\star = \pm\pi - \alpha$.

When there is no angle of attack we calculate that

$$(x,y,z) = (\mathscr{F}(z^\star) - r^\star\cos\theta^\star, -r^\star\sin\theta^\star, z^\star), \tag{3}$$

and hence the new coordinates are just cylindrical coordinates in the x-y plane, centred at $(\mathscr{F}(z^\star), 0)$.