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Trapped modes in a non-axisymmetric cylindrical waveguide

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ABSTRACT

We consider acoustic wave transmission in a non-axisymmetric waveguide which consists of a cylindrical resonator and two cylindrical waveguides whose axes are shifted relatively to each other by an azimuthal angle $\Delta\phi$. Under variation of the resonator's length L and fixed $\Delta\phi$ we find bound states in the continuum (trapped modes) due to full destructive interference of resonant modes leaking into the waveguides. Rotation of the waveguide adds complex phases to the coupling strengths of the resonator eigenmodes with the propagating modes of the waveguides tuning Fano resonances to give rise to a wave faucet. Under variation of $\Delta\phi$ with fixed resonator's length we find symmetry protected trapped modes. For $\Delta\phi \neq 0$ these trapped modes contribute to the scattering function supporting high vortical acoustic intensity spinning inside the resonator. The waveguide rotation brings an important feature to the scattering and provides an instrument for control of acoustic transmittance and wave trapping.

1. Introduction

Bound states in the continuum are localized solutions which correspond to discrete eigenvalues coexisting with extended modes of continuous spectrum in resonator-waveguide configurations. The existence of such modes was first reported Neumann and Wigner [1] at the dawn of quantum mechanics. To the best of our knowledge, the term *bound state (embedded) in the continuum* was introduced by Fonda [2] in the context of resonance reactions in the presence continuous channels. Since then bound state in the continuum (BIC) has been universally used to designate a state with discrete energy embedded into the continuum in quantum mechanics [3]. In the field of fluid mechanics, Parker [4,5] is credited to be the first to encounter resonances of pure acoustic nature in air flow over a cascade of flat parallel plates. Nowadays, BICs are known to exist in various waveguide structures [6–9]. BICs are of immense interest, especially, in optics thanks to experimental opportunity to confine light in optical microcavities despite the fact that outgoing waves are allowed in the surrounding medium [10–15].

Independently considerable attention has been paid to BICs in perturbed acoustic waveguides. Many different geometrical configurations with Neumann boundary conditions have been studied. These studies have shown that the existence of trapped modes is very sensitive to choice of geometry. Up to now geometrical configurations have been chosen to reduce the effective dimension of the acoustic waveguide. Chronologically, the following specific perturbed acoustic waveguides were considered. In 1951 Ursell [16,17] considered a sphere placed on the axis of a cylindrical guide and showed that a trapped mode exists for a selected radius of the sphere. There is a long history of trapped modes bound below the channel cut-off in two- and three-dimensional nonuniform waveguides due to the curvature of the waveguide or a localized bulge [18–21]. However the bound states with isolated discrete eigenvalue embedded in the continuous spectrum above the channel cut-off, BICs, are more unusual. Evans and Porter first provided convincing numerical evidence for BICs of both Neumann and Dirichlet types in the case of a

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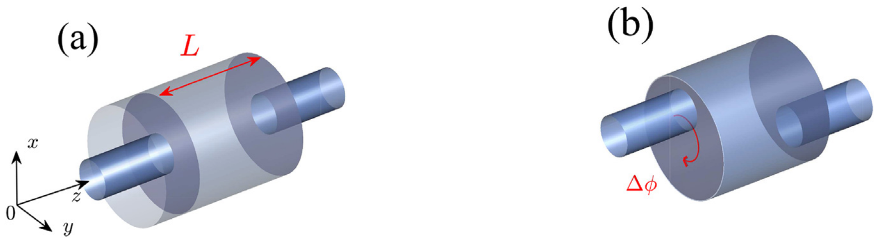

Fig. 1. Cylindrical resonator of radius R and length L with two attached cylindrical waveguides of the unit radius. All lengths are non-dimensional and measured in terms of the waveguide's radius r_w. The whole waveguide system is (a) axisymmetric, and (b) non-axisymmetric with waveguides misaligned by an azimuthal angle difference $\Delta\phi$.

rigid circular cylinder placed on the center-plane between parallel walls [22]. Linton and McIver [23] proved the existence of an infinite number of trapped modes for the case of a cylindrical waveguide containing an axisymmetric obstacle, in particular, a thin circular sleeve.

Similarly, the dimension is reduced in the acoustical waveguides of a rectangular cross-section in the $y0z$ plane and directed along the x-axis with an obstacle shaped only in the $x0y$ plane so that the thickness of the perturbed waveguide along the z-axis d is constant. Then the scattering channels are given by the eigenmodes quantized along the z-axis with corresponding Neumann boundary conditions at the walls positioned at $z = \pm d/2$. The utmost case of these structures is a two-dimensional acoustical waveguide formed by two infinite parallel lines at distance d containing a circle of radius $R < d$ [24] or multiple circles [25,26] positioned symmetrically between them. The trapped modes are antisymmetric about the centerline of the guide, which allows us to determine them as symmetry protected BICs. More sophisticated BICs of the same symmetry as the symmetry of the continuum were demonstrated recently in Refs. [27–30].

A different class is the fully three-dimensional systems. For example, in the case of non-axisymmetric obstacle inside the cylindrical waveguide Hein and Coch [31] numerically computed acoustic resonances and BICs by solving the eigenvalue problem. Here we consider similar non-axisymmetric waveguide but without an obstacle inside as shown in Fig. 1. The axisymmetric case shown in Fig. 1 (a) preserves the orbital angular momentum (OAM) m because of the rotational symmetry around the central axis. That effectively reduces the dimension of the waveguide to two. The BICs with $m = 0$ were shown to occur under variation of the length of the resonator [29] due to full destructive interference of resonant states [32]. An equivalent explication of BICs is that under variation of the resonator length, the eigenmodes ψ_1, ψ_2 of the same symmetry as the symmetry of propagating modes of the waveguides become degenerate. Then, the coupling of the superposed state $a_1\psi_1 + a_2\psi_2$ with the continuum can be cancelled by a proper choice of the superposition coefficients a_1 and a_2 [33]. In the present paper, we choose a different strategy for the trapping of acoustic waves by means of the rotation of one of the waveguides by the angle $\Delta\phi$ as shown in Fig. 1 (b). Then, one of the waveguides acquires azimuthal difference relative to the other that crucially affects interference of resonances, i.e., Fano resonances and the wave transmission. We show that even tiny rotations result in change of the transmittance from zero to unit, qualifying the setup as a wave faucet.

2. Acoustic coupled mode theory for open cylindrical resonators

There are different numerical approaches to calculate the transmittance through the waveguide with the mode matching method. The finite-element method with complex scaling is also widely exploited [28]. Here, we apply the method of effective non-Hermitian Hamiltonian [34–37] or, equivalently, the coupled mode theory, a physically transparent approach to diagnose BICs. The theory is based on the Feshbach projection technique [34] of the total space, resonator plus waveguides, onto the subspace of the resonator that results in the effective non-Hermitian Hamiltonian. Each subsystem possesses the rotational symmetry and obeys the stationary Helmholtz equation in the cylindrical system of coordinates

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{m^2}{r^2} + \frac{\partial^2}{\partial z^2} + \omega^2\right]\psi = 0, \tag{1}$$

for the non-dimensional velocity potential ψ, where the non-dimensional coordinates r and z express the respective distances normalized by the waveguide radius r_w. The dimensionless frequency ω is defined through the dimensional frequency $\widetilde{\omega}$ as follows $\omega = \widetilde{\omega} r_w/c_0$, and c_0 is the sound speed [28].

The propagating modes in the sound hard cylindrical waveguides with Neumann boundary conditions are described by

$$\psi_{pq}(\rho, \alpha, z) = \psi_{pq}(\rho)\frac{1}{\sqrt{2\pi k_{pq}}}\exp(\mathrm{i}p\alpha + \mathrm{i}k_{pq}z), \tag{2}$$

$$\psi_{pq}(\rho) = \begin{cases} \dfrac{\sqrt{2}}{J_0(\mu_{0q})}J_0(\mu_{0q}\rho), p = 0, \\ \sqrt{\dfrac{2}{\mu_{pq}^2 - p^2}}\dfrac{\mu_{pq}}{J_p(\mu_{pq})}J_p(\mu_{pq}\rho), p = 1, 2, 3, \dots, \end{cases}$$