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Issues in vibration energy harvesting

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ABSTRACT

In this study, fundamental issues related to bandwidth and nonlinear resonance in vibrational energy harvesting devices are investigated. The results show that using bandwidth as a criterion to measure device performance can be misleading. For a linear device, an enlarged bandwidth is achieved at the cost of sacrificing device performance near resonance, and thus widening the bandwidth may offer benefits only when the natural frequency of the linear device cannot match the dominant excitation frequency. For a nonlinear device, since the principle of superposition does not apply, the "broadband" performance improvements achieved for single-frequency excitations may not be achievable for multi-frequency excitations. It is also shown that a large-amplitude response based on the traditional "nonlinear resonance" does not always result in the optimal performance for a nonlinear device because of the negative work done by the excitation, which indicates energy is returned back to the excitation. Such undesired negative work is eliminated at global resonance, a generalized resonant condition for both linear and nonlinear systems. While the linear resonance is a special case of global resonance for a single-frequency excitation, the maximum potential of nonlinear energy harvesting can be reached for multi-frequency excitations by using global resonance to simultaneously harvest energy distributed over multiple frequencies.

1. Introduction

The performance of a vibrational energy harvesting device is often evaluated based on two parameters: bandwidth and resonance. Usually, they are determined by analyzing the frequency response of the device. Due to the principle of superposition, the frequency response curve serves as an effective assessment tool for linear devices. It shows that a linear device can perform well (i.e., efficiently) when the excitation is within a frequency range defined by the half-power bandwidth of the device. For ambient vibrations in which energy is distributed over a wide spectrum of frequencies or the dominating frequency is time-varying, linear devices become less efficient.

Because of the unique characteristics of nonlinear systems, attempts have been made in recent decades to improve device performance through intentionally introduced nonlinearity ([1,2]). In addition to the extensively studied Duffing-type nonlinearity ([3–7]), bi-stable systems have received considerable attention recently. The bi-stability can be achieved through different mechanisms, such as initially compressed beam ([8]), magnetic interaction ([9–13]), laminate material bi-stability ([14,15]) or buckling by the gravity ([16]). While the frequency response curve of a nonlinear device often demonstrates broader bandwidth compared to linear devices, it does not suggest the desired "broadband" performance for multi-frequency excitations because of the inapplicability of the principle of superposition. Results have shown that little improvement can be obtained through nonlinearity in a passive electromechanical device when it is driven by white noise base accelerations ([17–21]). The maximum

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power that can be harvested by a passive, memoryless device has been shown to depend only on the spectral density of the base acceleration and the total mass of the device ([20,22]). The presence of nonlinearity or multiple degrees of freedom does not affect this upper bound. For bi-stable nonlinear systems, however, it is possible to increase the harvested power by introducing a secondary forcing function that triggers stochastic resonance ([8,23]).

In addition, since mechanical energy is mostly harvested through damping ([24]), generating a large-amplitude response is intuitively beneficial. While increasing the response amplitude of a linear device is always advantageous, the same is not true for nonlinear devices. When a linear device is driven by a single-frequency excitation, the optimal performance occurs at resonance, i.e. when the frequencies of the excitation and the device are matched. The amplitude of the response and the amount of energy harvested are simultaneously maximized. In nonlinear cases, the resonant condition is not as simple. In fact, a generalized condition, referred to as global resonance, can be defined for both linear and nonlinear devices. Under this condition, the velocity response of the device is perfectly synchronized with the forcing function; there is no instantaneous phase difference between them. Therefore, the excitation always produces non-negative work. It has been shown that the response of a device (linear or nonlinear) at global resonance is identical to a free response of the undamped counterpart of the device ([25]). For linear devices under single-frequency excitations, this condition is equivalent to the well-known frequency match. For nonlinear devices, the global resonance is fundamentally different from the traditional concept of nonlinear resonance. The latter refers to the high-energy orbit of a harmonically excited nonlinear oscillator due to bifurcation [26]. The large amplitude resulting from nonlinear resonance is always accompanied by a phase difference between the velocity response and the forcing function, leading to the undesired negative work by the excitation. Under global resonance, however, the response may be lower but the harvested energy can be significantly higher because the negative work is eliminated as a result of the perfect synchronization of the device and the excitation ([25,27]).

In this study, therefore, fundamental issues with respect to bandwidth and nonlinear resonance are addressed. It is shown that using bandwidth as a performance criterion may be problematic for linear devices and it may be misleading when extended to nonlinear devices. To address the use of a large-amplitude response to enhance nonlinear energy harvesting, this study compares the device performance at the global resonance to that at the so-called nonlinear resonance, and thus reveal the real potential of nonlinear approaches in vibrational energy harvesting.

2. Governing equation

Consider an electromechanical energy harvesting device driven by a periodic excitation and assume that the electrical load of the device is purely resistive. The governing equation of the device dynamics can be generally written as

$$m\ddot{x} + c_m\dot{x} + \frac{\partial V(x)}{\partial x} + \kappa y = f(x,t)\ , \tag{1a}$$

$$\alpha\dot{y} + \beta y = \kappa\dot{x}\ , \tag{1b}$$

where x denotes the displacement of the seismic mass m, $\dot{(\ )}$ and $\ddot{(\ )}$ the first and second derivatives of the argument with respect to the time t, respectively, c_m the mechanical damping coefficient, $V(x)$ the potential well function of the system, κ the linear electromechanical coupling coefficient, and $f(x,t) = f(x,t+T)$ denotes the general external force with a fundamental period of T, including parametrical excitations (e.g., $f(x,t) = p(t)\sin x$ where $p(t)$ is the external excitation). For inductive devices, y, α and β represent the induced current, the winding inductance, and the total resistance, respectively. For capacitive ones, they represent the induced voltage, the capacitance of the piezoelectric element, and the load conductance, respectively. It is assumed that the energy of the excitations is concentrated on low frequencies and $\alpha << \beta$. Equation (1b) can be approximated as an algebraic relationship, i.e., $\beta y = \kappa\dot{x}$. The governing equation is thus simplified as

$$\ddot{x} + \gamma\dot{x} + \frac{\partial U(x)}{\partial x} = F(x,t)\ , \tag{2}$$

in which the total system damping coefficient is defined as $\gamma = \frac{c_m + \kappa^2/\beta}{m}$, and $U(x) = \frac{V(x)}{m}$, $F(x,t) = \frac{f(x,t)}{m}$.

3. Bandwidth

3.1. Linear system

Consider a linear device (i.e., $U(x) = \frac{1}{2}\omega_n^2 x^2$ where ω_n is the natural frequency) driven by a single-frequency base acceleration (e.g., $F(x,t) = A_e\cos\omega t$, where A_e is the acceleration amplitude). The average power per unit mass dissipated by damping is calculated to be

$$P_d = \frac{1}{T}\int_0^T \gamma\dot{x}^2 \mathrm{dt} = \frac{\frac{A_e^2}{2}\frac{\gamma}{\omega_n^2}r^2}{\left(1-r^2\right)^2 + \left(\frac{\gamma}{\omega_n}r\right)^2}\ , \tag{3}$$