Journal of Sound and Vibration 421 (2018) 91–110

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Use of the dynamic stiffness method to interpret experimental data from a nonlinear system

Bin Tang [a, b, *], M.J. Brennan [c], G. Gatti [d]

[a] Key Laboratory of Ocean Energy Utilization and Energy Conservation of Ministry of Education, Dalian University of Technology, Dalian, 116023, China
[b] Institute of Internal Combustion Engine, Dalian University of Technology, Dalian, 116023, China
[c] Departamento de Engenharia Mecânica, Universidade Estadual Paulista (UNESP), Ilha Solteira, SP 15385-000, Brazil
[d] Department of Mechanical, Energy and Managment Engineering, University of Calabria, Rende, CS, 87036, Italy

ARTICLE INFO

Article history:
Received 21 August 2017
Received in revised form 20 December 2017
Accepted 6 January 2018

Keywords:
Dynamic stiffness
Beam-shaker interaction
Compressed beam
Cubic stiffness

ABSTRACT

The interpretation of experimental data from nonlinear structures is challenging, primarily because of dependency on types and levels of excitation, and coupling issues with test equipment. In this paper, the use of the dynamic stiffness method, which is commonly used in the analysis of linear systems, is used to interpret the data from a vibration test of a controllable compressed beam structure coupled to a test shaker. For a single mode of the system, this method facilitates the separation of mass, stiffness and damping effects, including nonlinear stiffness effects. It also allows the separation of the dynamics of the shaker from the structure under test. The approach needs to be used with care, and is only suitable if the nonlinear system has a response that is predominantly at the excitation frequency. For the structure under test, the raw experimental data revealed little about the underlying causes of the dynamic behaviour. However, the dynamic stiffness approach allowed the effects due to the nonlinear stiffness to be easily determined.

1. Introduction

Modal analysis has become an important tool for the analysis of structural vibration measurements from controlled tests [1]. When carrying out such tests one of the most commonly used exciters is an electrodynamic shaker [2], which can have a significant influence on the resulting measurements. This has motivated research into shaker-structure interaction and into the factors affecting the dynamic behaviour of the shakers. Based on a physical model of a shaker, Lang and Snyder experimentally analyzed the performance of electrodynamic shaker [3,4]. More recently, the shaker parameters were estimated using the impedance approach [5,6], and a study of the causes of nonlinearity has been carried out by Saraswat and Nachiketa [7]. The criteria and challenges for the shaker in modal testing have been systematically summarized and discussed in Refs. [8–10].

One of the principal problems in vibration testing is the connection between the shaker and test structure. Mitchell and Elliott [11], Hu and McConnell [12–16], and Lee and Chou [17] have all analyzed the effects of stinger and force gauge on the estimation of the modal parameters of a structure. Mohammadali and Ahmadian [18] proposed a general analytical model to

* Corresponding author. Key Laboratory of Ocean Energy Utilization and Energy Conservation of Ministry of Education, Dalian University of Technology, Dalian, 116023, China.
E-mail address: btang@dlut.edu.cn (B. Tang).

https://doi.org/10.1016/j.jsv.2018.01.006
0022-460X/

Nomenclature

A	cross-section area of the beam
B	constant magnetic field strength
c	damping coefficient
E	Young's modulus of elasticity
$f(t)$	force generated by the shaker
f_n	$\omega_n/2\pi$
F	amplitude of $f(t)$
f_d	jump-down frequency
f_r	frequency at which the peak occurs in the displacement response of a damped linear oscillator
H_V	FRF of the complete system
H_F	FRF of the beam
I	current amplitude supplied to the shaker
k	linear stiffness coefficient
k_3	cubic stiffness coefficient
k_{b3}	cubic nonlinear stiffness for the beam
k_{sh3}	softening cubic stiffness for the shaker
l	length of the beam
$\boldsymbol{L}$	length of the coil
m	mass
t	time
T	transmissibility
V	voltage amplitude of the source
$w(y,t)$	displacement of the beam
$x(t)$	response of the beam
X	amplitude of $x(t)$
X_d	displacement amplitude of the response at the jump-down/maximum response frequency
X_{max}	maximum rated displacement of the armature
y	axis along the beam
Z	dynamic stiffness
α	relationship between voltage supplied to the amplifier and the blocked force generated by the shaker
ρ	mass per unit volume
ω	angular forcing frequency
ω_n	undamped natural frequency
ζ	damping ratio
ϕ	mode-shape

Subscripts

A	driving point
b	beam
B	mid-point of the beam
i	i-th mode
sh	shaker
total	combined beam-shaker system
(nonlinear)	nonlinear effect

investigate shaker-stinger-structure interaction. Hoffait et al. [19] developed a virtual shaker model to predict the dynamic characteristics of coupled shaker and test specimen. To minimize the attached mass effects of force gauge on lightweight structures, coil-in-magnet arrangements have been designed and used as excitation sources in Refs. [20–22].

Another principal problem in vibration testing is that of force "drop-out" at the resonance frequency of the test structure. This occurs because of the high mobility of the test structure at this frequency [23–25]. Dargah [26] systematically analyzed the interaction between the shaker and a structure, and the force "drop-out" in vibration testing. Whilst this is not a particular problem for the majority of tests for linear structures, it can be problematic in the testing of nonlinear structures [27], because the dynamics of a nonlinear structure depend on the amplitude of excitation, unlike that of a linear structure. Because of the increasing importance of nonlinearity in modern structures, the interaction between the shaker and the structure in vibration testing needs to be understood, and there is scant literature in this area. There is, however, some related studies on nonlinearity in electrodynamic shakers. The analysis of force distortion due to the nonlinear characteristics of electrodynamic