Journal of Sound and Vibration 420 (2018) 330–345

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Band structure analysis of a thin plate with periodic arrangements of slender beams

Ó. Serrano *, R. Zaera, J. Fernández-Sáez

Department of Continuum Mechanics and Structural Analysis, University Carlos III of Madrid, Av. de la Universidad 30, 28911 Leganés, Madrid, Spain

ARTICLE INFO

Article history:
Received 8 March 2017
Revised 30 October 2017
Accepted 9 November 2017
Available online 3 February 2018

Keywords:
Lattice structure
Beam arrays
Bloch's theorem
Band gap
Dimensional analysis

ABSTRACT

This work analyzes the wave propagation in structures composed of a periodic arrangement of vertical beams rigidly joined to a plate substrate. Three different configurations for the distribution of the beams have been analyzed: square, triangular, and hexagonal. A dimensional analysis of the problem indicates the presence of three dimensionless groups of parameters controlling the response of the system. The main features of the wave propagation have been found using numerical procedures based on the Finite Element Method, through the application of the Bloch's theorem for the corresponding primitive unit cells. Illustrative examples of the effect of the different dimensionless parameters on the dynamic behavior of the system are presented, providing information relevant for design.

1. Introduction

The design and use of structures able to promote directivity in wave propagation, or even to cancel it at certain frequencies ranges (band gaps) have attracted a great interest in several scientific and technological fields. It is well known that periodic solids, which can be treated as lattice structures, provide the above characteristics.

Among the great variety of periodic solids, structures composed by an array of parallel nanobeams located normal to a nanoplate (substrate) have been used in several applications related to nanosensors, water photoelectrolysis, hydrogen storage devices and gas sensing [1,2].

The dynamic behavior of this kind of system has been analyzed by several authors (Collet *et al.* [3], Tsung-Tsong *et al.* [4], Tzung-Chen *et al.* [5] and Zi-Gui *et al.* [6]). The above works focus on periodic stubbed surfaces where short beams with circular [3–5] or square-cross sections [6] are distributed in a square arrangement over the surface. The predicted band structures have been verified with experimental results [4,5]. Tsung-Tsong *et al.* [4] and Tzung-Chen *et al.* [5] showed the corresponding band structures for different ratios of beam length to plate thickness, while Zi-Gui *et al.* [6] studied the influence on the first band gaps of the rotated angle of the square cross-section of the stubs around its longitudinal axis. These analyses are rather specific because only one configuration (square arrangements of the stubs) and the variation of only one parameter is considered.

Other authors focus on a kind of structure with a higher beam-length to plate-thickness ratio. Tanaka *et al.* [7] showed some characteristics of these kinds of structures, in which a polymer surrounds the arrangement of beams. Eremeyev *et al.* [8,9] conducted an analytical study of the dynamic behavior of a system composed by ZnO nanocrystals (slender nanobeams) clamped to a sapphire substrate (nanoplate). It was found that the eigenfrequencies of the system can be determined as a combination of the eigenfrequencies of a single nanocrystal and those of the substrate [8]. Elsewhere a dense continuous distribution of beams

* Corresponding author.
E-mail address: oscar.serrano@uc3m.es (Ó. Serrano).

https://doi.org/10.1016/j.jsv.2017.11.016
0022-460X/

over the substrate is presented [9].

The control of location and width of the band gaps is a key design issue of this type of structure in order to achieve the desired performance related to a specific application. Sugino et al. [10] studied the case of a plate with a square arrangement of resonators (masses and springs vibrating perpendicularly to the plate). These authors finished with the evidence of a band gap located at the resonant frequency of the spring-mass resonator and the width depending on the ratio of resonator mass to plate mass. This technological interest stimulates the proposal of methodologies for *optimal* design, aiming at selecting the parameters of the system that can position the band gap within a certain range. Some authors explored the use of topology optimization as a systematic way to design phononic band-gap materials [11,12]. In topology optimization, a given design layout is discretized by a large number of elements, commonly coinciding with the FE mesh, allowing the material type or density in each element to be treated as a design variable. By defining objective functions, the above-cited authors determine the characteristics of the solid for the maximization of the band gaps. Despite the interest of this methodology to define entirely new topologies tailored for specific loads and boundary conditions, the overall picture of the physical behavior of the solid frequently remains hidden. Moreover, the presence of multiple optima, i.e. non-unique solutions, introduces uncertainties in the design process. Here we follow an alternative pathway, grounded in the knowledge of the dimensionless groups governing the solution to the problem. This enables us to specify which parameters are relevant (and which are not), how they influence the band structure, and where the global extrema are located.

In this work, we analyze the dynamic behavior of a periodic array of slender beams normal to a thin substrate, considering it as a lattice structure. This periodicity permits the use of Bloch's methodology. On the basis of the formulation by Eremeyev *et al.* [9], we develop a dimensional analysis which reveals three dimensionless groups of variables governing the dynamic behavior of the lattice. Three different geometrical distributions for the crystal are considered (square, triangular, and hexagonal). For each configuration an analysis of band structures and directivity of wave propagation is shown. Also, illustrative results on the influence of the different dimensionless groups of variables on the location and width of the band gaps are presented.

The paper is organized as follows. Section 1 provides a brief introduction and Section 2 describes the problem and the geometries involved in the study. Section 3 offers the dimensional analysis of the problem while Section 4 describes the methodology used to apply Bloch's theorem together with the Finite Element Method on the different primitive unit cells. Section 5 presents some illustrative examples on the wave-propagation characteristics of the three studied lattice structures. Section 6 illustrates the influence of the different dimensionless parameters in the behavior of the system. Finally Section 7 summarizes the main results of the work.

2. Problem formulation

Let us consider a system consisting of an infinite plate parallel to the plane $\{x, y\}$, and an infinite array of beams perpendicular to the plate and with their lower ends joined to it. The assumed constraint condition at the beam-plate interface is that of equal rotations and out-of-plane displacement. The plate has surface density ρ and bending stiffness D. The beams have length l, linear density ρ_* and the same bending stiffness C in both directions. Let (U, V, W) be displacements and $\left(\theta_x, \theta_y, \theta_z\right)$ rotations, the degrees of freedom of the system are W, θ_x, θ_y for the plate, and U, V, W, θ_x, θ_y for the beams. Fig. 1 illustrates the degrees of freedom considered. Moreover, the beams are considered inextensible. As stated above, thin plate and slender beams are considered, thus permitting the use of the Kirchhoff and Euler-Bernoulli models respectively, neglecting the effect of shear strains.

The position of the beams is assumed to follow a periodic arrangement in the $\{x, y\}$ plane, thus constituting a lattice structure. Three different arrangements are studied: square, triangular, and hexagonal, with connection points located at the vertices of

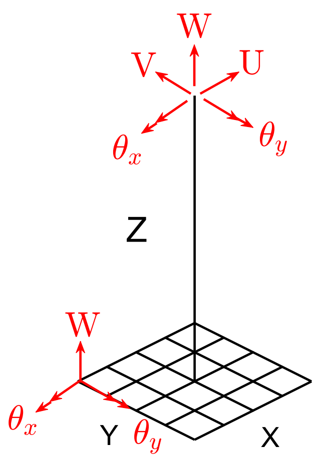

Fig. 1. Scheme of a lattice structure with the degrees of freedom considered.