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Nonlinear analysis of a rotor-bearing system using describing functions

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ABSTRACT

This paper presents a technique for modelling the nonlinear behavior of a rotor-bearing system with Hertzian contact, clearance, and rotating unbalance. The rotor-bearing system is separated into linear and nonlinear components, and the nonlinear bearing force is replaced with an equivalent describing function gain. The describing function captures the relationship between the amplitude of the fundamental input to the nonlinearity and the fundamental output. The frequency response is constructed for various values of the clearance parameter, and the results show the presence of a jump resonance in bearings with both clearance and preload. Nonlinear hardening type behavior is observed in the case with clearance and softening behavior is observed for the case with preload. Numerical integration is also carried out on the nonlinear equations of motion showing strong agreement with the approximate solution. This work could easily be extended to include additional nonlinearities that arise from defects, providing a powerful diagnostic tool.

1. Introduction

Rolling element bearings are a key source of vibration in rotor bearing systems, creating a demand for accurate vibration analysis and diagnostic techniques under a variety of operating conditions. The rotor-bearing system exhibits nonlinear behavior due to bearing clearance, nonlinear Hertzian contact force, and defects. In the presence of rotating unbalance, which is unavoidable in a rotor-bearing system, a wide variety of system behavior can be expected as a result of nonlinearities in the system. Capturing this nonlinear behavior in parametric models that are computationally efficient and sufficiently accurate is important for a variety of applications, including diagnostics and modelling of more complicated rotating systems with multiple bearings. This study focuses on the nonlinear analysis of a rotor-bearing system with clearance, rotating unbalance, and nonlinear Hertzian contact using a simplified mathematical representation of the bearing force. These characteristics are all present in a healthy bearing and therefore necessary to establish base-line behavior as a function of system parameters.

The vibration analysis of rotor-bearing systems has received considerable attention, using both linear and nonlinear models. Clearance, which is usually designed into rolling element bearings to compensate for thermal expansion, has been studied extensively. Yamamoto is credited with some of the earliest work [1] characterizing the response of a shaft supported by bearings with radial clearance. In this work it was shown that the maximum amplitude at critical speed tends to decrease with increasing radial clearance. Mevel and Guyader [2] reported different routes to chaos in a bearing with clearance including sub-harmonic

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and quasi-periodic routes, using numerical integration of nonlinear equations of motion. Additionally, "loss of contact" was identified as a necessary condition to allow chaotic motions. Since the bearing system is inherently nonlinear, numerical integration has become a popular method for studying nonlinear bearing system behavior. [3–13] studied the vibration response of rigid rotors supported by rolling element bearings with radial clearance using numerical techniques. Various nonlinear phenomena such as chaos, bifurcations, quasi-periodicity, and sub-harmonics have been reported in this work. Additionally, these researchers all report a decrease in the likelihood of chaos as clearance decreases and more erratic or unstable behavior for larger values of clearance. The effect of flexible raceways in a rotor bearing system with radial clearance was studied by both Leblanc et al. [14] and Gupta et al. [15]. Leblanc et al. reported a significant change in ball pass harmonics with the addition of flexible rings. Gupta et al. reported more pronounced effects of the clearance nonlinearity with an increase in stiffness ratio, resulting in a higher likelihood of chaos and instability. Kappaganthu and Nataraj [16] studied an alternative rotor-bearing system model that does not include the assumption of collinear raceways. Limit cycles, quasi-periodic oscillations, and chaos were all reported.

Other researchers have simplified the rotor-bearing system in order to utilize linear analysis techniques. Ghafari et al. [17] used a Taylor Series approximation about a single equilibrium point to derive an analytical solution. A softening effect was observed for increases in radial clearance as well as a decrease in the natural frequency. Liew et al. [18] developed a time averaged bearing stiffness model where the bearing stiffness is estimated by a time-varying stiffness that fluctuates about the mean stiffness. The authors reported strong agreement between the time-varying stiffness model and direct numerical integration.

Several researchers have studied the rotor-bearing system with radial clearance using a nonlinear analysis technique known as the harmonic balance method [19–21] where the nonlinear system response is estimated as a truncated Fourier series. Tiwari et al. [22] studied the effect of radial internal clearance on the vibration response using a modified harmonic balance method. The peaks in the vibration response were shown to shift down with increase in clearance, and decreases in clearance were shown to increase the linear characteristics of the system. Sinou [23] also used the harmonic balance method with a condensation process on the nonlinear degrees of freedom to estimate the nonlinear unbalance response for a flexible rotor-bearing system with clearance. A softening type nonlinearity was observed by increasing the radial clearance. The peak in the unbalance response was also shown to decrease in frequency with increase in clearance. Jump phenomena were observed during run-up and run-down with constant operation. These types of system characteristics are difficult to capture using a linearized system or numerical integration.

Most of these previous studies utilized numerical integration techniques to solve the bearing system equations of motion. Numerical techniques are inefficient and make parametric studies cumbersome. Additionally, nonlinear phenomena such as jump resonance are difficult or impossible to capture, especially in the region of jump where multiple solutions exist. An approximate tool which is simple enough to give insight into the trends of system behavior as a function of system parameters is often more advantageous than a complicated exact analytical tool. This is particularly true for analysis of rotor-bearing systems as clearance and Hertzian contact are both nonlinear components intrinsic to the system. The harmonic balance method offers an approximate tool, but the Fourier coefficients are typically found using numerical techniques such as Broyden's method or may even require the exact solution from numerical integration. The focus of this work is the use of a much simpler technique that preserves nonlinear phenomena.

In this paper, the describing function method is utilized to model the nonlinear bearing force and construct the frequency response for a rotor-bearing system with clearance. The derived equations of motion are non-dimensionalized for parametric studies of system behavior. The linear and nonlinear portion of the system are separated and a sinusoidal-input describing function is used to represent the nonlinearity that arises from the bearing force. The bearing force is therefore replaced with an equivalent gain that is dependent upon the input amplitude and system parameters. This method of handling the bearing nonlinearity significantly simplifies the computation required to characterize the many facets of nonlinear system behavior.

For a nonlinear element subject to a sinusoidal input, the describing function captures the relationship between the amplitude of the fundamental input and the fundamental output [24]. In this work, the rotor displacement is the input to the nonlinearity and the output is the bearing force. With the assumption of a sinusoidal input to the system resulting from rotating unbalance, the nonlinear bearing force is replaced with an equivalent gain. Since the describing function gain only deals with the relationship between the fundamental input and output, any harmonics or distortion are neglected in analysis. It is assumed that such harmonics are sufficiently filtered by the linear portion of the system, making the approach agree reasonably well with the nonlinear system. This is known as the "filter hypothesis" in describing function analysis. In the case of the rotor-bearing system with unbalance, higher harmonics of ball-pass frequencies are assumed to be sufficiently filtered by the linear part of the system.

Using the derived describing function for the bearing force, the frequency response is constructed for various levels of clearance and preload. The techniques used for constructing the frequency response are similar to those from linear system analysis. Many researchers have reported the nonlinear phenomena associated with the rotor-bearing system adopted for this work. The goal of this work is to validate the accuracy of the proposed technique and treatment of the bearing nonlinearity, which has not been carried out in any of the previous work. Values of the system parameters are sought that generate nonlinear phenomena such as jump resonance, and the results are confirmed with numerical integration. A secondary objective is to provide a better understanding of healthy bearing behavior by studying the frequency response for key system parameters.