Journal of Sound and Vibration 419 (2018) 405–419

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

A wavenumber approach to analysing the active control of plane waves with arrays of secondary sources

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ARTICLE INFO

Article history:
Received 8 June 2017
Revised 21 November 2017
Accepted 10 January 2018
Available online XXX

Keywords:
Active noise control
Feedforward control
Wavenumber domain

ABSTRACT

The active control of an incident sound field with an array of secondary sources is a fundamental problem in active control. In this paper the optimal performance of an infinite array of secondary sources in controlling a plane incident sound wave is first considered in free space. An analytic solution for normal incidence plane waves is presented, indicating a clear cut-off frequency for good performance, when the separation distance between the uniformly-spaced sources is equal to a wavelength. The extent of the near field pressure close to the source array is also quantified, since this determines the positions of the error microphones in a practical arrangement. The theory is also extended to oblique incident waves. This result is then compared with numerical simulations of controlling the sound power radiated through an open aperture in a rigid wall, subject to an incident plane wave, using an array of secondary sources in the aperture. In this case the diffraction through the aperture becomes important when its size is compatible with the acoustic wavelength, in which case only a few sources are necessary for good control. When the size of the aperture is large compared to the wavelength, and diffraction is less important but more secondary sources need to be used for good control, the results then become similar to those for the free field problem with an infinite source array.

1. Introduction

The active control of an incident sound field inside a closed surface using a discrete array of secondary sources has been considered using numerical models in the 2-D case of a circle by Zavadskaya et al. [1] and in the 3-D case of a sphere by Konyaev et al. [2]. The secondary sources were assumed to be combinations of point monopoles and dipoles, as described by Mangiante [3] for example as a Huygens' source, regularly arranged to control the sound inside the surface without external radiation. It was found in both the 2-D and the 3-D studies [1,2] that the spacing between the secondary sources, required to achieve a good level of control within the closed surface needed to be closer than about $\lambda/2$, where λ is the acoustic wavelength, as noted by Nelson and Elliott [4].

It might be expected that as the radius of the surface becomes very large, the problem would reduce to that of the control of an incident plane wave by an infinite array of secondary sources normal to the direction of propagation. It is shown below, however, that perfect control of such a normally-incident plane wave can be achieved with an infinite array having a separation of λ, rather than $\lambda/2$. This analysis is best performed using a wavenumber decomposition of the secondary source array and subsequent sound radiation, and this wavenumber analysis can be extended to plane waves that are not normal to the array,

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https://doi.org/10.1016/j.jsv.2018.01.028

both in 2-D and in 3-D.

Although it is of fundamental interest to analyse the limits to the performance of such an array, it potentially has applications in sound reproduction and in the active control of environmental noise. It becomes more difficult to analyse the performance when the size of the array is finite, although this case is applicable to the active control of sound transmitted through open windows, as considered, for example, by Murao and Nishimura [5], Lam *et al* [6] and Wang *et al* [7]. The analytic results for an infinite array are therefore compared with the results from a numerical simulation of active control in this application.

A preliminary version of this paper with a less general formulation for the active control was presented by Elliott *et al* [8], and some of the numerical simulations in Section 6 were presented by Bhan *et al* [6].

2. Control of a normally-incident plane wave with an infinite array of secondary sources in 2-D

In order to easily illustrate the method, we begin by considering the active control of a normally-incident plane wave in free space using an infinite 2-D array of secondary sources. This arrangement is illustrated in Fig. 1a, where the objective is to suppress the incident primary wave to the right of the array of line sources in the plane of figure. If the aim was to actively absorb the incident wave, each of the secondary sources would need to be a combination of a monopole and a dipole, arranged such that the sound radiation to the left of the array was minimised. The problem to the right of the array, however, is similar to that if the secondary sources were only assumed to be monopoles, although in this case the secondary source array would reflect the incident wave and so the sound field to the left of the array would consist of a standing wave, due to the interference between the incident and a reflected wave, as it does for a plane wave in a duct controlled with a single secondary source [4]. In order to provide generality, and extension to the control of oblique waves in Section 4, a wavenumber analysis of the problem is presented here, although in an earlier conference publication [8] the control of a normally-incident plane wave was analysed using an analogous duct formulation [9].

We assume a primary plane wave of the form

$$p_p(z) = p_p \mathrm{e}^{-\mathrm{j}k_0 z}, \tag{1}$$

where z is the direction normal to the array, as shown in Fig. 1 k_0 is ω/c_0, c_0 is the speed of sound and p_p is the complex pressure proportional to $\mathrm{e}^{\mathrm{j}\omega t}$. The normal particle velocity associated with this wave is

$$v_p(z) = \frac{p_p}{\rho_0 c_0} \mathrm{e}^{-\mathrm{j}k_0 z}, \tag{2}$$

where ρ_0 is the density of the medium. To control this primary field we assume an infinite array of secondary line sources, uniformly separated by a distance d in the x direction, which must all have equal source strengths, by symmetry. If each secondary source strength per unit length is q_s', then the normal particle velocity associated with the secondary sources when z is equal to zero is

$$v_s(x, 0) = q_s' \sum_{m=-\infty}^{\infty} \delta(x - md), \tag{3}$$

where δ is the Dirac delta function and m is the secondary source index. Taking the Fourier series of $v_s(x, 0)$ gives

$$v_s(x, 0) = \frac{q_s'}{d} \sum_{n=-\infty}^{\infty} \mathrm{e}^{\mathrm{j}2\pi nx/d}, \tag{4}$$

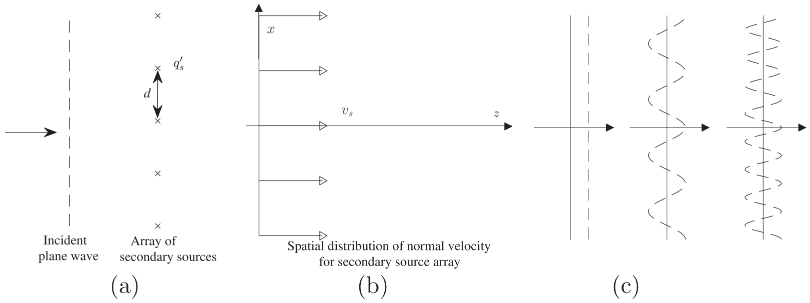

Fig. 1. Physical arrangement in 2-D of an incident plane wave being controlled to the right of an infinite array of secondary line sources in the plane of the figure (a). Also shown (b) is the spatial distribution of the normal velocity for a section of the secondary source array and the first three spatial harmonics of its wavenumber decomposition (c).