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Hybrid time-variant reliability estimation for active control structures under aleatory and epistemic uncertainties

Lei Wang [a,*], Chuang Xiong [a], Xiaojun Wang [a], Yunlong Li [b], Menghui Xu [c]

[a] Institute of Solid Mechanics, Beihang University, Beijing, 100191, China
[b] Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801, United States
[c] Faculty of Mechanical Engineering & Mechanics, Ningbo University, Ningbo, Zhejiang 315211, China

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ABSTRACT

Considering that multi-source uncertainties from inherent nature as well as the external environment are unavoidable and severely affect the controller performance, the dynamic safety assessment with high confidence is of great significance for scientists and engineers. In view of this, the uncertainty quantification analysis and time-variant reliability estimation corresponding to the closed-loop control problems are conducted in this study under a mixture of random, interval, and convex uncertainties. By combining the state-space transformation and the natural set expansion, the boundary laws of controlled response histories are first confirmed with specific implementation of random items. For nonlinear cases, the collocation set methodology and fourth Rounge-Kutta algorithm are introduced as well. Enlightened by the first-passage model in random process theory as well as by the static probabilistic reliability ideas, a new definition of the hybrid time-variant reliability measurement is provided for the vibration control systems and the related solution details are further expounded. Two engineering examples are eventually presented to demonstrate the validity and applicability of the methodology developed.

1. Introduction

Vibration issues are inevitable problems in structural analysis and design. Because the demands for elimination of structural vibrations are increasing and conventional technologies based on the passive vibration control have difficulty meeting the design requirements, the approaches of active vibration control have gradually gained more attention and practical experience in the fields of aeronautic and aerospace engineering [1–3]. Actually, the active control system is an organic combination of sensors to measure structural responses, actuators to impose auxiliary forces, and control law to analyze how the control forces are applied [4]. Therefore, general active control problems may involve multi-disciplinary categories, including materials science, structural dynamics, signal processing, control principle, etc [5].

Nevertheless, with the wide application of smart materials and complicated structures in aerospace industry, the safety problems that exist in active control systems are becoming more prominent [6]. Different from the conventional treatments of passive control, the main characteristic of the active control lies is the continuous external energy input that is applied to

* Corresponding author.
E-mail address: leiwang_beijing@buaa.edu.cn (L. Wang).

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the real structure in accordance with the current state feedback [7]. Hence, tiny fluctuations in parameters may lead to significant deviations in the aspect of control effects, and may even directly cause system instability [8,9].

Indeed, the uncertain factors of controlled structures are notable, impersonal and ubiquitous. In recent years, with the deep recognition of uncertainty analysis in structural mechanics, the reliability estimation of active vibration control systems has aroused widespread public concern as well [10,11]. In the 1990s, Venini et al. introduced the concept of reliability into active control systems based on a probabilistic model, in which the reliability index was first defined as the safety criteria for the effectiveness of the controller [8]. Spencer et al. employed first and second order reliability methods (FORM/SORM) to calculate the probabilistic reliability measures of a controlled structure, and the stability of the closed-loop control system was regarded as the limit-state constraint [12,13]. Battaini et al. considered the stability conditions of fuzzy control schemes under random wind excitation and parameter uncertainties, and the safety estimation of the controlled structures can be achieved via reliability measurements derived from SORM approximations [14]. Taflanidis et al. deployed a theoretical analysis of reliability-based design optimization for active structural control applications, and the controller was designed by minimizing the probability of structural failure [15].

As revealed by the abovementioned literature, most of the existing works were performed in the probabilistic frame, which needs to construct probability density functions (PDFs) for quantifying uncertain variables by using substantial amounts of sample data. However, in practical engineering, multi-source uncertainties, which mainly consist of material dispersion, geometry deviation, loads fluctuation, signal noises, etc., have always existed in actual control systems, and in most cases, the uncertainty information is extremely insufficient [6,16]. Under such circumstances, the preceding probability reliability approaches will no longer be eligible, and thus, the non-probabilistic reliability analytic methods, especially the hybrid reliability methods should be further developed. Additionally, current studies are mainly focused on the static/time-invariant reliability evaluation, but the time dependency of vibratory responses is ignored when tackling reliability issues.

In terms of the non-probabilistic reliability methods, the original work was carried out by Ben-Haim and Elishakoff in the early 1990s [17,18], and further developed by Qiu et al. [19,20], Jiang et al. [21,22], and Du et al. [23,24] in the past two decades. Guo employed the infinity norm of the uncertain variables vector to define the stability reliability of controlled structures [16,25]. Li et al. provided a systematic non-probabilistic reliability analysis procedure for structural vibration active control system, in which its solution index was obtained by the ratio of the volume of the safe region to the total feasible one [5,26]. Over the past few decades, in the fields of structural time-variant reliability, the first-passage method based on Rice's formula has been recognized as the most popular method [27]. According to the random process theory, Crespo and Kenny [28], Chakraborty and Roy [29] accomplished the reliability analysis and design for uncertain control systems by solving the out-crossing rate. Other methodologies included the joint-upcrossing rate method, the asymptotic method, the Markov Chain method, and the Monte Carlo simulation method, which are also summarized in Refs. [30–32].

Despite this, the study on how to simultaneously consider the multi-source uncertainties and time-dependency into the reliability assessment of active vibration control problems is still in its preliminary stage. This paper aims to develop a new hybrid time-variant reliability method for vibration active control systems with mixed aleatory and epistemic uncertainties. The reminder is organized as follows. Section 2 discusses classical state-space transformation and the reduced-order model for solving the vibration active control issues. In section 3, the mixed uncertainties under randomness, interval, and convex are taken into account to compute the uncertain responses for close-loop control structures. By utilizing the uncertain response results, section 4 defines the hybrid time-variant reliability index and details its solution procedure. Section 5 conducts an in-depth analysis of feasibility of the proposed methodology in aspects of control equation calculation and time-discretization treatments. Then, two numerical examples are provided in section 6 to demonstrate the effectiveness of the present method, followed by some conclusions.

2. Solution strategies for deterministic vibration active control problems

2.1. General governing equation of structural state-space transformation

Consider the following dynamic system with n degrees of freedom modeled using the finite element method, which is commonly adopted in the vibration control

$$\begin{cases} \boldsymbol{M}\ddot{\boldsymbol{w}}(t) + \boldsymbol{P}\dot{\boldsymbol{w}}(t) + \boldsymbol{K}\boldsymbol{w}(t) = \boldsymbol{B_s}\boldsymbol{u}(t) + \boldsymbol{B_e}\boldsymbol{f}(t) \\ \boldsymbol{w}(t_0) = \boldsymbol{w}_0, \dot{\boldsymbol{w}}(t_0) = \dot{\boldsymbol{w}}_0 \end{cases} \tag{1}$$

where $\boldsymbol{M} \in \mathbb{R}^{n \times n}$, $\boldsymbol{P} \in \mathbb{R}^{n \times n}$, and $\boldsymbol{K} \in \mathbb{R}^{n \times n}$ are the mass, damping and stiffness matrices, respectively; $\boldsymbol{w}(t), \dot{\boldsymbol{w}}(t), \ddot{\boldsymbol{w}}(t) \in \mathbb{R}^{n \times 1}$ stand for the response vectors of displacement, velocity and acceleration, respectively; the matrices $\boldsymbol{B_s} \in \mathbb{R}^{n \times r}$ and $\boldsymbol{B}_e \in \mathbb{R}^{n \times s}$ reflect the position and quantity information of the input control forces $\boldsymbol{u}(t) \in \mathbb{R}^{r \times 1}$ and the external loads $\boldsymbol{f}(t) \in \mathbb{R}^{s \times 1}$; $\boldsymbol{w}(t_0)$ and $\dot{\boldsymbol{w}}(t_0)$ denote the initial conditions.

By introducing the state vector $\boldsymbol{x}(t) = [\boldsymbol{w}^T(t), \dot{\boldsymbol{w}}^T(t)]^T$, one can rewrite Eq. (1) as a usual first-order state-space form [3,8], as defined by