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Design and analysis of fractional order seismic transducer for displacement and acceleration measurements

Parthasarathi Veeraian, Uma Gandhi, Umapathy Mangalanathan*

Department of Instrumentation and Control Engineering, National Institute of Technology, Tiruchirappalli, India

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Seismic transducers are widely used for measurement of displacement, velocity, and acceleration. This paper presents the design of seismic transducer in the fractional domain for the measurement of displacement and acceleration. The fractional order transfer function for seismic displacement and acceleration transducer are derived using Grünwald–Letnikov derivative. Frequency response analysis of fractional order seismic displacement transducer (FOSDT) and fractional order seismic acceleration transducer (FOSAT) are carried out for different damping ratio with the different fractional order, and the maximum dynamic measurement range is identified. The results demonstrate that fractional order seismic transducer has increased dynamic measurement range and less phase distortion as compared to the conventional seismic transducer even with a lower damping ratio. Time response of FOSDT and FOSAT are derived analytically in terms of Mittag-Leffler function, the effect of fractional behavior in the time domain is evaluated from the impulse and step response. The fractional order system is found to have significantly reduced overshoot as compared to the conventional transducer. The fractional order seismic transducer design proposed in this paper is illustrated with a design example for FOSDT and FOSAT. Finally, an electrical equivalent of FOSDT and FOSAT is considered, and its frequency response is found to be in close agreement with the proposed fractional order seismic transducer.

1. Introduction

The concept of fractional calculus, whose history begins in the 17th century, has gained considerable importance during the past four decades, many researchers and scientists have contributed to the development and application of fractional calculus. Fractional Calculus was motivated first by Oldham and Spanier [1]. Miller and Ross [2], Samko et al. [3] made valuable contributions to the theory of fractional integration and differentiation. The extensive use of fractional calculus in many applications developed after Podlubny [4] giving the geometrical and physical interpretation of fractional integration and differentiation. Fractional differentiation with respect to time can be interpreted as an existence of memory effects which correspond to intrinsic dissipation in the system [5]. In recent time fractional calculus has been extensively used by the researchers for modeling physical systems associated with dynamic viscoelastic materials [6]. The concept of fractional calculus has been applied to various fields including biomedical applications [7]- [8], agriculture [9], electromagnetics [10], electrical circuits [11–13], control design [14], development of fractional order capacitors based on electrolyte processes [15],

* Corresponding author.
E-mail addresses: sarathii.veera@gmail.com (P. Veeraian), guma@nitt.edu (U. Gandhi), umapathy@nitt.edu (U. Mangalanathan).

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modeling of lossy coils [16] and constant phase elements [17], filters [18], chaotic systems [19], impedance matching network [20], transmission line [21] and system identification [22–23].

Seismic transducers operate on the principle of measuring the motion relative to that of inertial mass. Through appropriate design of the mass-spring-damper system, the output is a direct indication of either displacement or acceleration in the specified frequency range [24]. The seismic transducer used to detect motion has considerable industrial significance with a wide range of applications in aerospace, terrestrial and marine transportation. Accelerometers are commonly used in airbag restraint systems [25], measuring low-frequency and low-amplitude structural vibration [26], for digital navigation and control systems [27]. Other applications include biomedical [28] and electrical motors [29]- [30].

The methods of classical mechanics deal only with conservative systems, while almost all processes observed in the physical world are non-conservative and exhibit irreversible dissipative effects which can be better modeled by fractional calculus [4]. Blair suggested fractional-order models of viscoelastic behavior and interpreted separate time scale for different materials based on the observation, subjective judgments of the time do not follow the Newtonian time scale [4]. Chakraverty and Behera [31] have applied homotopy perturbation method for a fractionally damped single degree-of-freedom spring-mass-damper system and obtained their dynamic responses. An oscillatory processes model associated with damping processes with a fractional damping term was modeled, and the response was obtained by applying the fractional differential equation theory, by Torvik and Bagley [32]. Gómez-Aguilar et al. [33] presented fractional differential equations for the mass-spring-damper system and showed that the mechanical components exhibited viscoelastic behaviors producing temporal fractality at different scales and demonstrated the existence of material heterogeneities in the mechanical components, which characterizes the existence of fractal structures. Most recently, Naranjani et al. [34] designed, distributed order fractional damper for a simple mechanical oscillator to meet the objective function that includes peak-overshoot, peak time and the integrated tracking error. Di Paola et al. [35] presented equivalent mechanical models constituted by linear dashpots and springs in appropriate arrangements, for a fractional operator.

Inspired by the above works in this article we present a fractional order seismic transducer and analyze the output of the transducer for displacement and acceleration measurements. Section 2 introduces the fundamental concepts of fractional calculus; Section 3 describes the mathematical model of proposed fractional order seismic transducer. The analysis of seismic transducer for displacement and acceleration measurement is carried out in Section 4, 5 and corresponding subsections. In the subsequent sections, the proposed fractional order transducer is demonstrated with a design example and its electrical equivalent. And Section 7 concludes this work. The work presented in this paper is first of its kind which uses fractional calculus for analyzing the dynamic characteristics of a seismic transducer.

2. Preliminary to fractional calculus

There are number of definitions in the literature for fractional differentiation, of which Riemann–Liouville (RL), Riesz, Weyl, Grünwald–Letnikov (GL), and the Caputo derivative were extensively used. In studying the characteristics of the fractional order seismic transducer, the major concern is analyzing its frequency response. In such context GL fractional derivative [3] given in Eq. (1) is normally used.

$$\mathrm{D}_l^{\alpha} f(t) = \lim_{h\to 0^+} \frac{\sum_{n=0}^{\infty} (-1)^n \binom{\alpha}{n} f(t-nh)}{h^{\alpha}}; \alpha > 0 \tag{1}$$

For the above-defined derivative

$$\mathrm{D}^{\alpha} \mathrm{e}^{\mathrm{i}\omega \mathrm{t}} = (\mathrm{i}\omega)^{\alpha} \mathrm{e}^{\mathrm{i}\omega \mathrm{t}}. \tag{2}$$

This facilitates to define frequency response. The Laplace transform of GL fractional derivative of order $\alpha > 0$ with zero initial condition is given by Eq. (3),

$$L\{{}_0\mathrm{D}_t^{\alpha} f(t)\} = s^{\alpha} F(s) \tag{3}$$

Fundamental physical considerations in favor of the use of models based on derivatives of non- integer order are given in Ref. [36]. In order to keep the dimensionality of the physical equations unchanged, an auxiliary parameter τ is introduced in the fractional operator, the unit of τ is second, this parameter is associated with the temporal components of the system [33], [36]. The fractional derivative is expressed as,

$$\frac{\mathrm{d}^{\gamma}}{\mathrm{d}t^{\gamma}} = \frac{1}{\tau^{n-\gamma}} {}_0\mathrm{D}_t^{\gamma}; n-1 < \gamma < n, \tag{4}$$

where n is an integer.

Two parameter Mittag-Leffler function for $t > 0$ is given by Eq. (5) [37],