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Constitutive error based parameter estimation technique for plate structures using free vibration signatures

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ABSTRACT

In this paper, a variant of constitutive equation error based material parameter estimation procedure for linear elastic plates is developed from partially measured free vibration signatures. It has been reported in many research articles that the mode shape curvatures are much more sensitive compared to mode shape themselves to localize inhomogeneity. Complying with this idea, an identification procedure is framed as an optimization problem where the proposed cost function measures the error in constitutive relation due to incompatible curvature/strain and moment/stress fields. Unlike standard constitutive equation error based procedure wherein a solution of a couple system is unavoidable in each iteration, we generate these incompatible fields via two linear solves. A simple, yet effective, penalty based approach is followed to incorporate measured data. The penalization parameter not only helps in incorporating corrupted measurement data weakly but also acts as a regularizer against the ill-posedness of the inverse problem. Explicit linear update formulas are then developed for anisotropic linear elastic material. Numerical examples are provided to show the applicability of the proposed technique. Finally, an experimental validation is also provided.

1. Introduction

Dynamic characteristics like eigen-frequencies, mode shapes, and modal damping of existing structures change due to the deterioration of structural properties locally i.e. the presence of local damages. Thus, changes in dynamic response can be effectively used for detection of local damages. This detection procedure comprises of locating the position as well as quantification of damages. Structural damage detection techniques involving identification of heterogeneous material properties belong to the class of inverse problem wherein material properties are obtained from full or partially (possibly corrupted) measured data. Among dynamic characteristics based damage detection techniques, modal response based techniques gained more attention due to its direct linkage with physical quantities like natural frequencies and mode shapes of a structure.

Modal data based damage identification has been rapidly expanded over the last few decades and many inversion techniques can be found in existing literature. A comprehensive overview on these methods can be found in Refs. [1,2]. In Ref. [3], it is suggested that curvature of mode shapes are highly sensitive to localize structural damage and thus can be used to locate and quantify the size of the damage effectively. A new correlation coefficient, Multiple Damage Location Assurance Criterion (MDLAC) has been introduced in Ref. [4] to find the location and sizes of multiple damages using the information of changes of modal properties for damaged and undamaged state of the structure. In Ref. [5], modal curvature based damage detection technique is proposed for simply supported beam and concrete bridge structure. Curvature damage factor (CDF) has

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been introduced here for the detection of multiple damages. An efficient damage detection technique was developed in Ref. [6] using the changes of frequency response function (FRF) of the modal filter using a large network of sensors. Effectiveness of slope of mode shapes in detecting localize damage has been shown in Ref. [7] by detecting localize damages for steel plates with different boundary effects. It has been observed that damages can be perfectly identified with a single lower frequency data. It is also observed that mode shapes curvatures are stable far beyond from the damaged region and henceforth there is no need of fine grid measured mode shapes data for damage detection. Artificial neural network technique was used effectively in detecting location and severity of damage for simply supported beam using mode shape and modal strain energy in Ref. [8]. An enhanced 2D modal curvature based damage detection technique for isotropic plate structure was illustrated in Ref. [9].

Identification of anisotropic material parameters, in comparison to isotropic case, is difficult due to the increased number of material constants. Moreover, heterogeneous (localized) nature of damage makes the identification problem much more difficult because of the large number of unknown material parameters. Thus, instability and non-uniqueness of the ill-posed problem plays a crucial role for the success of any inverse procedure. With the advancement of measurement technology, several attempts have been made to identify homogeneous anisotropic material parameters [10–12]. For example, in Ref. [13], it is observed that curvature of mode shapes can be used successfully to detect the damages in E-glass/epoxy composite laminated plate structure. In Ref. [14], a polynomial annihilation edge detection technique is developed to detect damage for plate like structures based on mode shape data. The method was applied to detect delamination for laminated composite plate with numerically simulated measured mode shapes data.

Generally damage detection problems are attempted via minimizing the least-squared error of measured and computed dynamic responses (e.g. natural frequency, mode shape) with most well established sensitivity [15,16] based approach. A quasi-Newton (gradient based), Gauss-Newton or full-Newton method can be used depending upon the computational cost [17,18]. However, least square (L_2) based estimation techniques often faces numerical instabilities due to their sensitivity to the initial guess and usually take a large number of iterations to obtain the desired reconstruction. Another technique for elastic parameter estimation was proposed based on the concept of error in the constitutive equation (ECE) in terms of an energy norm. The idea of error in constitutive equation (ECE), also known as constitutive relation error (CRE), was developed in Refs. [19–21] for posteriori error estimation in finite element approximation. Physical interpretation of this type of cost function is more meaningful due to its direct linkage with constitutive relation. A modified version of ECE known as MECE (also known as modified version of CRE i.e. mCRE) was introduced in heterogeneous identification elastic parameters in transient dynamics [22,23]. The MECE or mCRE functional is robust with respect to noise and brings more flexibility compared to ECE alone in which measurements data would be enforced in the definition of admissibility. An effective method of reducing computational cost in finite element model updating technique using mCRE was proposed in Ref. [24]. Constitutive equation gap method (CEGM), a variant of ECE was used in isotropic heterogeneous material parameter estimation procedure using static displacement data by Refs. [25,26]. Also, MECE based identification technique has been extended for elastoplastic parameter [27,28], rate dependent material parameter [29], hyperelastic material parameter [30]. The ECE based constitutive compatibility method (CCM) was formulated for heterogeneous identification of linear elastic material parameter by Refs. [31] and [32]. It was shown in Ref. [33] that MECE method needs very few iterations when compared to standard least square approach for large scale frequency domain elasto-dynamic problems.

Recently, MECE method has also been explored for identification of heterogeneous linear elastic material parameters using numerically simulated free vibration data [34]. One of the main challenges of the MECE based approach is the generation of two incompatible moment and curvature fields which are coupled in nature. Nevertheless to say, for large scale problem, the computational cost is very high for solving such a coupled system. It may also be noted here that successive over-relaxation (SOR) technique, as proposed in Ref. [33], is not applicable for free vibration problem. However, techniques such as domain decomposition [32], model reduction [35] procedure can be effectively used to reduce the computation cost of solving the coupled system. In this context, an efficient model reduction technique based on proper generalized decomposition (PGD) was proposed in Ref. [35] in reducing cost of ECE (or CRE) based model updating procedure. The objective of this paper is to bypass the solution of the coupled system for parameter estimation within the standard MECE framework. In doing so, we have explored a new variant of error in constitutive equation based material parameter identification procedure that does not require to solve a coupled system for material parameter estimation.

This article is organized as follows. In section 2 we have briefly described the free-vibration problem of plate structure following the Reissner-Mindlin assumptions. Then, standard least-squares and the MECE based approaches for parameter identification are briefly touched upon. The proposed constitutive error based inverse material parameter estimation technique is demonstrated in section 3. Here we have posed parameter identification problem into a minimization problem of the cost functional with two incompatible moment and curvature fields. Unlike the MECE approach, the proposed formulation needs to solve two linear forward problems. Then explicit material update equations are developed for isotropic and orthotropic materials. A short discussion on choice of penalty parameters is also given. In section 4, numerical experiments are given. Experimental validation of the proposed technique is also explored in section 5. Finally conclusions are drawn in section 6.