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Inelastic interaction and splitting of strain solitons propagating in a rod

V.I. Erofeev [a, b], V.V. Kazhaev [a, b], I.S. Pavlov [a, b, *]

[a] Mechanical Engineering Research Institute of the Russian Academy of Sciences, 85, Belinsky Str., 603024, Nizhny Novgorod, Russia
[b] Nizhny Novgorod Lobachevsky State University, 23, Gagarin Av., 603950, Nizhny Novgorod, Russia

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The results of original research of the soliton interaction and the interaction of solitons with a boundary of a rod are presented in this paper. Moreover, the paper contains a brief analytical review of publications devoted to the theoretical study of the formation and peculiarities of propagation of longitudinal strain solitons in nonlinearly-elastic rods, as well as publications devoted to the experimental detection of such waves.

1. Introduction

From the theory of waves it is known that in one-dimensional systems the entire variety of wave processes is determined by the relationship between nonlinearity, dispersion, and dissipation. In two-dimensional and three-dimensional systems, the phenomenon of diffraction is added to them. In the case, when nonlinear, dispersion and diffraction effects compensate each other and if dissipation is small, solitary nonlinear stationary waves (solitons) propagating with a constant velocity and conserving their shape can arise in the system. According to the definition given in Ref. [1], “Soliton is a structurally stable solitary wave in a nonlinear dispersive medium. Solitons behave like particles: when they interact with each other and with some other perturbations, solitons do not collapse, but diverge again, keeping their structure unchanged.”

The term “soliton” was introduced by N. Zabusky and M. Kruskal in 1965 [2], but this type of wave was noted yet in 1834 by J. Scott Russell, when he observed waves in channels. The soliton solution for long waves on the surface of a liquid was first obtained by Boussinesq (1872). In 1895, Korteweg and de Vries derived an equation called by their names and found its solution in the form of periodic (cnoidal) waves. C. Gardner, J. Greene, M. Kruskal, and R. Miura developed the inverse scattering problem method (ISPM) in 1967 [3] and discovered some completely integrable equations [4]. After that, the interest to solitons continuously grew.

As attention to solitons increased, since the 1970s, interest has grown in non-linear waves in a rod, because it is one of the most available objects for experimental studying and, at the same time, it is widely used in engineering [5–7]. A rod is usually considered as a deformable solid body possessing finite rigidity for tension, torsion and bending, and two sizes of which are

* Corresponding author. Mechanical Engineering Research Institute of the Russian Academy of Sciences, 85, Belinsky Str., 603024, Nizhny Novgorod, Russia.

E-mail address: ispavlov@mail.ru (I.S. Pavlov).

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small in comparison with the third one. Normal waves are divided in rods into three types: longitudinal, torsional and bending ones. In rectilinear rods, different types of waves do not interact in the linear approximation [8]. Various scenarios of inelastic soliton interaction, which have been obtained by numerical simulation depending on the relative velocity of interaction, are given in the present paper.

2. Basic mathematical models describing propagation of nonlinear longitudinal solitary waves (solitons) in a rod

The classical D. Bernoulli model (the technical theory) assumes that for describing the longitudinal vibrations of a rod it is possible to neglect the potential energy of shear deformations and the kinetic energy of the transverse motions of rod particles. According to this theory, the linear waves propagate in the rod with velocity $c_0 = \sqrt{E/\rho}$ (here E is the Young's modulus of elasticity and ρ is the density of a material) that does not depend on frequency. Consequently, these assumptions do not allow describing the geometric dispersion of longitudinal waves, which is observed in experiments [1].

The mathematical models proposed by Rayleigh and Love, Bishop, Mindlin, and Herrmann (improved theories) [5] eliminate this shortcoming. The Bishop and Mindlin-Herrmann models are those one-dimensional models, which most accurately describe the dispersion of longitudinal waves in a wide frequency range.

Account of the nonlinearity of the relationships between strains and displacement gradients (geometric nonlinearity), strains and stresses (physical nonlinearity) leads to nonlinear generalizations of the mathematical models mentioned above.

Under certain conditions, the Korteweg-de Vries equation (KdV equation for short) is suitable for description of nonlinear longitudinal waves in a rod:

$$v_t + 6vv_x + v_{xxx} = 0, \tag{1}$$

where $v = u_x$ and u is a longitudinal displacement of the particles of the midline, x is a dimensionless coordinate, t is a dimensionless time; subscripts t and x mean derivatives with respect to time and coordinate, accordingly. This fact was first shown by G.A. Nariboli in Ref. [9].

Later, dissipative effects were taken into account and the Burgers-Korteweg-de Vries equation (BKdV) was derived by G.A. Nariboli and A. Sedov in their joint work [10]:

$$v_t + \alpha vv_x - \beta v_{xx} - \gamma v_{xxx} = 0. \tag{2}$$

Here α, β, and γ are the parameters describing influence of nonlinearity, dissipation and dispersion. Eq. (2) describes the influence of dispersion and dissipation on waves of small but finite amplitude in viscoelastic rods. It should be noted that in Eqs. (1) and (2) and further in the text, the letter subscript denotes differentiation with respect to the corresponding independent variable.

In Refs. [9,10], the KdV equation was obtained from the exact set of the elasticity theory equations describing nonlinear waves in a circular rod by the perturbation method with respect to several small parameters.

L.A. Ostrovsky and A.M. Sutin [11] considered the propagation of longitudinal waves in a homogeneous rod made from a nonlinearly elastic material, which internal energy is a function of the invariants of the strain tensor with accuracy up to the cubic term. They showed that an evolution of the longitudinal component of the displacement vector is described by the equation, which is a nonlinear generalization of the Rayleigh-Love model or the nonlinear Boussinesq equation:

$$u_{tt} - c_0^2\left(1 - \frac{6\alpha}{E}u_x\right)u_{xx} - \sigma^2 r_p^2 u_{xxtt} = 0.$$

Here $\alpha = \frac{E}{2} + \frac{\nu_1}{6}(1-6\sigma) + \nu_2(1-2\sigma) + \frac{4}{3}\nu_3$ is the coefficient characterizing the geometric and physical nonlinearities of the rod; σ is Poisson's ratio; r_p is the polar radius of inertia of the cross section of the rod; ν_1, ν_2, and ν_3 are Lame constants of the third order.

The method of simplifying of sets of equations with small nonlinearity and dispersion developed by one of the authors was applied to the obtained equation [12]. In the case of small nonlinearity and dispersion, this equation is reduced to the KdV Eq. (1).

In the work of V.I. Erofeev and A.I. Potapov [13], a technique was proposed for reducing the three-dimensional equations of the nonlinear elasticity theory to approximate equations of the theory of rods. This technique is based on the approximation of displacements in the cross section of a rod and the application of the Hamilton's variational principle. Nonlinear equations generalizing the Bishop and Mindlin-Herrmann models have been obtained. The first of these equations has the form:

$$u_{tt} - c_0^2\left(1 - \frac{6\alpha}{E}u_x\right)u_{xx} - \sigma^2 r_p^2\left(u_{tt} - c_\tau^2 u_{xx}\right)_{xx} = 0, \tag{3}$$

where $c_\tau = \sqrt{\mu/\rho}$ is the elastic shear wave velocity in a material; μ is a shear modulus.

In Refs. [14,15] A.M. Samsonov and E.V. Sokurinskaya researched in detail Eq. (3) rewritten in terms of the displacement gradients: