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Modal density of rectangular structures in a wide frequency range

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ABSTRACT

A novel approach to investigate the modal density of a rectangular structure in a wide frequency range is presented. First, the modal density is derived, in the whole frequency range of interest, on the basis of sound transmission through the infinite counterpart of the structure; then, it is corrected by means of the low-frequency modal behavior of the structure, taking into account actual size and boundary conditions. A statistical analysis reveals the connection between the modal density of the structure and the transmission of sound through its thickness. A transfer matrix approach is used to compute the required acoustic parameters, making it possible to deal with structures having arbitrary stratifications of different layers. A finite element method is applied on coarse grids to derive the first few eigenfrequencies required to correct the modal density. Both the transfer matrix approach and the coarse grids involved in the finite element analysis grant high efficiency. Comparison with alternative formulations demonstrates the effectiveness of the proposed methodology.

1. Introduction

The modal density of a system is a frequency dependent function defined as the number of modes which lie in a unitary frequency interval. Its knowledge is required in the medium-high frequency range, when the number of modes make inapplicable the modal analysis. In such frequency range the response of a structure under mechanical or acoustic excitation can be deduced from its modal density. The modal density of various structures has been identified both theoretically and experimentally. For common structural components, such as a thin plate, beams, a spherical cap or a circular cylinder, the modal density was established through analytical expressions [1–4]. More complex configurations have also been studied, *e.g.* an expression for the modal density of honeycomb panels with orthotropic face sheets including transverse shear effects was established under specific hypothesis [5]. Parametric studies have also been performed in order to investigate the influence of various parameters on the modal density of a sandwich panel [6]. However, the analysis of general structures is often difficult.

The most commonly used procedure for deriving the modal density of a structure involves solving the dispersion problem for free-wave propagation in the structure. The mathematics of wave propagation in periodic systems was first discussed by Brillouin [7] in the field of electrical engineering. Afterwards, Orris and Petyt [8,9] employed the Finite Element (FE) technique for wave propagation analysis in periodic structures. Modal density depends on the group and phase velocity of the wave group considered. Langley [10] derived the modal density of periodically stiffened beam and plate structures in terms of phase constants associated with propagating wave motion. Finnvedan [11] used the wave-guide FE method to calculate the wave propagation characteristics of built-up thin-walled structures; he described the process of deriving the modal density and group

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velocity from FE input for a beam structure. However, the free-wave approach is reliable at high frequency only, since it considers structures with an infinite extent. Moreover, obtaining the modal density of a structure with several wave groups requires the intervention of the analyst to identify and discriminate dispersion curves.

Alternatively, a modal analysis of the finite structure with real boundary conditions can provide the modal density by means of the mode count. However, a modal approach is viable only at low frequency because of the related computational effort, regardless of the numerical method adopted.

A procedure to evaluate the modal density of a planar, rectangular structure in a wide frequency range is proposed. The key idea on the basis of the present work is the separation of the stacking and the boundary effects on the panel modal density. Such a separation allows to combine the ability of a dispersion problem to catch high-frequency structure dynamics, and the flexibility of a modal analysis in describing boundary effects at low frequency. The original contribution of the present work is twofold: first, an expression relating the modal density of an arbitrarily stratified, planar structure with the diffuse transmission and reflection coefficients of its infinite counterpart is presented, and second, a corrective scheme for the modal density is derived, taking into account the real size and boundary conditions of the structure by means of its low-frequency modal behavior. Involving sound transmission through the structure thickness instead of the dispersion problem makes it possible to bypass any difficulty related to dispersion curves. A Transfer Matrix Method (TMM) is used to evaluate the required acoustic indicators, allowing to deal efficiently with structures with generic stratifications, possibly including in-plane periodic layers [12]. The correction accounting for real size and boundary conditions requires only few eigenfrequencies of the structure, making it possible to stem the computational cost related to the modal analysis.

Section 2 presents the separation of the stacking and the boundary effects on the panel modal density. The stacking contribution is derived in Section 3 by means of a Statistical Energy Analysis (SEA) on sound transmission through the structure. Section 4 reveals the role of the low eigenfrequencies in correcting the modal density. A number of applications are then discussed and compared with alternative formulations.

2. Overview

The scope of the present work is twofold: first, to define the modal density of a planar structure avoiding the solution of the dispersion problem and second, to exploit the modal approach at low frequency in order to effectively take boundary effects into account. The key idea consists in treating stacking and boundary effects independently. To clarify this we consider the example of a simply supported, homogeneous, rectangular thin plate with dimensions $a \times b \times h$. The modal density of the plate can be expressed as [13]

$$n(\omega) = \frac{S}{4\pi}\sqrt{\frac{m}{B}} - \frac{P}{8\pi\sqrt{\omega}}\sqrt[4]{\frac{m}{B}}, \tag{1}$$

where $S = ab$ is the panel area, $P = 2a + 2b$ is the panel perimeter, $m = \rho h$ is the mass per unit area, $B = \frac{Eh^3}{12(1-\nu^2)}$ is the flexural rigidity, E is Young's modulus, ν is the Poisson ratio and ρ is the density. The expression for the modal density, Eq. (1), contains information about the stacking properties of the panel, through B and m, and the panel dimensions, through S and P. In order to bring out such a dichotomy, the asymptotic specific modal density is introduced:

$$\mu_\infty(\omega) = \lim_{S\to\infty} \frac{n(\omega)}{S}. \tag{2}$$

Since boundary effects quickly vanish moving away from panel edges, and so becoming null in a panel with infinite extent, the asymptotic specific modal density, μ_∞, depends only on the stacking properties of the panel. Such an interpretation is crucial for the proposed methodology, since it allows an evaluation of μ_∞ considering an infinite extent for the panel. This, in turn, allows the use of both a free-wave approach, *i.e.* TMM, and a statistical approach, *i.e.* SEA, in the whole frequency range of interest, even at low frequencies. Invoking the expression of the modal density obtained for the thin plate, Eq. (1), we obtain

$$\mu_\infty(\omega) = \frac{1}{4\pi}\lim_{S\to\infty}\left(\sqrt{\frac{m}{B}} - \frac{P}{2S\sqrt{\omega}}\sqrt[4]{\frac{m}{B}}\right) = \frac{1}{4\pi}\sqrt{\frac{m}{B}}. \tag{3}$$

It should be noted that the dependency on the frequency disappears in the obtained expression of the specific modal density, Eq. (3), because shear deformation is neglected in the adopted thin plate model. Dependency on the frequency persists in general and is contemplated by the proposed methodology. Using the expression for the specific modal density, Eq. (3), the modal density, Eq. (1), may be expressed as

$$n(\omega) = S\mu_\infty(\omega) - \frac{P}{4}\sqrt{\frac{\mu_\infty(\omega)}{\pi\omega}}, \tag{4}$$

where the way in which the panel dimensions act on the specific modal density, μ_∞, is highlighted. Even though the obtained expression for the modal density, Eq. (4), is valid for a thin plate (shear deformation neglected in the kinematic model) with simply supported boundary conditions, it can be seen as a reliable way of separating asymptotic and boundary contributions to the modal density of a generic panel. In fact, even a simple kinematic model, *e.g.* the thin plate model, can accurately catch