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Generalized topology for resonators having N commensurate harmonics

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ABSTRACT

Despite the ubiquity of both linear and nonlinear multimember resonators in MEMS and kinetic energy harvesting devices very few research efforts examine the orientation of members in the resonator on its dynamic behavior. Previous efforts to design this type of resonator constrains the members to have relative orientations that are 0° or 90∘ to each other, i.e., the elements are connected inline with adjoining members or are perpendicular to adjoining members. The work expands upon the existing body of research by considering the effect of the relative orientation between members on the dynamic behavior of the system. In this manuscript, we derive a generalized reduced-order model for the design of a multimember planar resonator that has integer multiple modal frequencies. The model is based on a Rayleigh Ritz approximation where the number of degrees of freedom equals the number of structural members in the resonator. The analysis allows the generation of design curves, representing all the possible solutions for modal frequencies that are commensurate. The generalized model, valid for an N-DOF structure, is then restricted for a 2- and 3-DOF system/member resonator, where the linear dynamic behavior of the resonator is investigated in depth. Furthermore, this analysis demonstrates a rule of thumb; relaxing restrictions on the relative orientation of members in a planar structure, allows the structure to exhibit exactly N commensurable frequencies if it contains N members.

1. Introduction

Tailoring the dynamic response of a structure based on its shape or topology can be a daunting task. However, this has been of particular interest to the design of resonators for kinetic energy harvesters and MEMS based resonators. Efforts in this area are: 1) ad hoc relying on structures with geometries that historically have been observed to exhibit certain dynamic behavior [1,2], 2) rely on shape optimization routines that have restrictions on the geometry of the resonator [3–6], or 3) use techniques from topology optimization where the optimal distribution of material is determined computationally [7–12].

In designing resonators that have 1:2 modal frequency ratio, the first approach has a long history in the research community. A common geometric configuration of resonators exhibiting this behavior are L-shaped structures and frames with perpendicular members [6,12–14]. L-shaped resonators have been of interest to the dynamics community for several decades [15–18]. This structure can behave effectively as 2-DOF system with commensurate frequencies [16]. When these resonators are excited in a nonlinear regime, they exhibit rich dynamic behavior by having: 1) strong modal interactions resulting in an energy exchange between the modes when the system is excited, i.e., an internal resonance [13,14,19,20], 2) saddle node [21,22], cyclic-fold [23]

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List of symbols

A	Cross sectional area of beam member
$a_{j,i}$	Generalized coordinates
b	Width of beam member
c	Constant
$\widehat{d}$	Nondimensional CG distance
E	Young's modulus
F, f	Free node set, free node
$\widehat{\mathbf{F}}, \widehat{\mathbf{f}}$	Generalized Force vector
G	Base excitation
h	Beam's height
I	Area moment of inertia
$\widehat{\mathbf{K}}$	Stiffness matrix
L	Beam's length
$\mathcal{L}$	Lagrangian
$\widehat{\mathbf{M}}$	Mass matrix
N	Number of commensurable frequencies (DOF of the structure)
n_{beams}	Number of structural members
p, q	first and second node of each member
T	Kinetic energy
V	Potential energy
V^*	Actual volume
V_g	Volume of the initial ground structure
W^*	Local bending displacement
w	Global bending displacement
u	Global axial displacement
v	Global total displacement
X_g, Y_g	CG coordinates
β	set of members that share an internal node
γ	Volume fraction
$\overline{\epsilon}_{xx}, \epsilon_{xx}$	Average and maximum strain
ζ	Damping coefficient
Λ	Frequencies ratio
Θ, θ	Internal node set, internal node
λ_γ	Weight function
μ	Mass per unit length
ξ, η	Local coordinate system
ρ	Density
φ	Folding angle with respect to the global reference
Ω	Design domain
$\omega, \widetilde{\omega}$	Bending frequencies, nondimensional bending frequencies
$\dot{()}$	Time derivative
$()_{,\xi}, ()_{,\xi\xi}$	First and second spatial derivatives

and Hopf bifurcations [24], and 3) saturation, i.e, the amplitude of a mode being directly excited becomes independent of the level of excitation [25].

In designing vibration energy harvesters (VEHs) the L-shaped structure's internally resonant behavior has been exploited improve the range of frequencies that energy can be extracted, [19,20]. Harne *et. al* [26] exploited the saturation phenomena present in the L-shaped structure to enhanced the average power scavenged by the device.

Concurrently, VEHs have been designed with orthogonally oriented members, essentially a collection of L-shaped resonators. These designs either: 1) maximize the power response [27], 2) in a fixed volume a lower bending stiffness, an increase in mass or both [28–30]. The latter makes the devices suitable for harvesting energy at low frequencies.

Reviewing research efforts that use techniques from structural optimization, several studies are noteworthy. Tripathi and Bajaj [6] presented a computational synthesis based on FEA to achieve natural frequencies of a structure in some desired ratio, notably 1:2 and 1:3. These planar, orthogonally connected multi-member structures are intended for MEMS applications. Deng et al. [31] used topology optimization to maximize the frequency response in the design of vibration energy harvesting electro-magnetic device. Furthermore, Dou et al. [12] presented a formulation based on shaped optimization to tailor the nonlinear response of a resonator that considers multiple modes in the response.

This work here begins with the review of a topology optimization study of resonators with 1:2 and 1:3 modal frequencies ratios. The optimization shows that the historically observed L-shape structures are not the only configuration that allows this dynamic behavior and the relative orientation of members in also important in determining the modal characteristics of the structure. The remainder of the paper is organized as follows: Section 2 briefly recaps a topology synthesis of commensurate resonators based on a ground structure and a linear FE analysis. The optimization takes the form of a multi-objective function is such that it minimizes the number of beams in the design space. Section 3, motivated by the results of the optimization, presents reduced order model capable of identify all the possible solutions to have commensurable frequencies in a mulitmember planar structure. The structural members of the model is based on Euler-Bernoulli beam assumption. The scope of this work is to demonstrate that, by relaxing the constraint on the angle among two contiguous beams, is possible to enlarge the design space and obtaining simpler geometries and alternative solutions to the ones known to date. Furthermore, it is proven that a rule of thumb holds, i.e. that a N-DOF planar system can be designed such that it has exactly N commensurable frequencies.

2. Motivation through topology synthesis

In view of the discussion presented in the previous section, Danzi *et. al* [32] sought to design resonators with integer multiple natural frequencies using finite element analysis coupled with a topological optimization routine based on a ground structure (GS) or structural universe parameterization of the design domain. Their findings serves as the primary motivation for considering the relative orientation in the design of resonators with integer ratio frequencies. Here, we briefly summarize their approach

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