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A quasi-one-dimensional theory of sound propagation in lined ducts with mean flow

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ABSTRACT

Sound propagation in ducts with locally-reacting liners has received the attention of many authors proposing two- and three-dimensional solutions of the convected wave equation and of the Pridmore-Brown equation. One-dimensional lined duct models appear to have received less attention. The present paper proposes a quasi-one-dimensional theory for lined uniform ducts with parallel sheared mean flow. The basic assumption of the theory is that the effects of refraction and wall compliance on the fundamental mode remain within ranges in which the acoustic fluctuations are essentially uniform over a duct section. This restricts the model to subsonic low Mach numbers and Helmholtz numbers of less than about unity. The axial propagation constants and the wave transfer matrix of the duct are given by simple explicit expressions and can be applied with no-slip, full-slip or partial slip boundary conditions. The limitations of the theory are discussed and its predictions are compared with the fundamental mode solutions of the convected wave equation, the Pridmore-Brown equation and measurements where available.

1. Introduction

Two well-known wave equations of the convected type for sound propagation in a homogeneous inviscid fluid are the convected wave equation and the Pridmore-Brown equation, which differ essentially by whether the mean flow velocity is assumed to be uniform or sheared. Solutions of these equations for linear wave modes in uniform lined ducts have received the attention of large number of authors. The convected wave equation admits well-understood and clear-cut solution procedures. However, the Pridmore-Brown wave equation [1], is not of the regular Sturm-Liouville type [2,3] and it appears from previous work that it is not tractable to a general analytical treatment, except in some special cases [3,4].

Various methods have been proposed for the solution of the convected wave equation and the Pridmore-Brown equation for lined ducts and two articles presenting extensive reviews of the previous work are available [5,6]. Most of the contemporary studies are motivated by turbofan engine research, where the Helmholtz numbers of interest are about $k_o a \approx 10$ or higher ($k_o = \omega/c_o$ denotes the wavenumber, ω the radian frequency, c_o the speed of sound of the fluid and a denotes a characteristic dimension of the duct cross-section). However, lined ducts have potential uses also in fluid machinery intake and exhaust systems where Helmholtz numbers of about $k_o a \approx 1$ or less may be of importance. In this low Helmholtz number range, the fundamental (lowest) mode usually predominates as the least attenuated mode and is sufficient in engineering design considerations. The present paper is concerned about predicting the fundamental mode propagation at such low Helmholtz numbers by using a one-dimensional model.

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One-dimensional analyses of sound propagation in ducts are ubiquitous in the literature, though lined ducts do not seem to have received attention in more recent publications. A one-dimensional wave equation for a homogeneous lined duct with no mean flow is presented in 1944 by Molloy [7] by adding ad hoc the effect of wall compliance on small volumetric fluctuations of an elementary fluid column in the classical plane wave formulation. Molloy states that this wave equation is equivalent to the transmission line model developed earlier by Sivian [8], who also carried out attenuation measurements on a circular duct lined with rock-wool and showed that the model is adequately accurate for $k_o a < 1.5$. However, a one-dimensional model for sound propagation in lined ducts with mean flow is not available. Such a model is an interesting alternative to the solution of the convected wave equation and the Pridmore-Brown equation for the fundamental mode, as it can be used to get reasonably accurate predictions for the propagation constants quickly.

The lined duct model proposed in the present paper is a quasi-one-dimensional one (subsequently referred to as Q1D model for brevity). It is based on the integral forms of the general 3-D continuity and momentum equations of inviscid fluid dynamics. These equations are reduced to differential forms in 1-D by using the divergence theorem and averaging the fluid properties over a duct cross-section. A crucial feature of this procedure, which makes the way for explicit application of the impedance condition at the duct walls, is the retaining of the component of the particle velocity normal to the duct walls in the formulation. The continuity and momentum equations are linearized as usual, the mean flow being assumed to be homogeneous fully developed turbulent grazing flow over smooth walls. In addition to the usual no-slip and full-slip mean flow models, a partial-slip model in which the acoustic boundary condition at the duct walls is deferred to the border of the viscous sublayer is also considered. In the latter model, we borrow from the results of Starobinski [9] for the estimation of the sublayer border impedance. For zero mean flow, the wave equation of the proposed Q1D lined duct model reduces to Molloy's [7] wave equation (Section 2.7).

The fundamental mode characteristics of the convected wave equation and the Pridmore-Brown equation have been studied by many authors, but usually in the context of high Helmholtz numbers that are relevant to aircraft applications. To our knowledge, the only paper reporting comparison of the fundamental mode predictions for Helmholtz numbers in the range $ka < 1$ with measurements is that by Tack and Lambert [10]. They investigate the sound propagation in a rectangular duct symmetrically lined on two opposite sides in the low Helmholtz number range and compare the measured and theoretical results obtained from the solutions of the convected wave equation with full-slip and the Pridmore-Brown equation with no-slip. We present a comparison of the predictions of the present Q1D lined duct model with the results of Tack and Lambert [10].

As already stated, most of the published solutions of the convected wave equation and the Pridmore-Brown equation for lined ducts are on aircraft applications and focus on results at relatively high Helmholtz number ranges, but there is an interesting exception. Lined duct design for aircraft industry requires accurate knowledge of the liner impedance with grazing mean flow and measurement, or impedance eduction as it is commonly known, is normally the only way to obtain this information. Test facilities used for impedance eduction essentially consist of a rectangular duct which has solid walls except over a finite length of one side, which is treated with the liner to be tested. The data gathered in impedance eduction tests usually pertain to the least attenuated mode of propagation. The present paper includes a comparison, based on the measurement data published by Renou and Aurégan [11], of the impedance spectra educed by using the proposed Q1D lined duct model, with the results educed by using the fundamental mode solution of the convected wave equation with full-slip. Also presented, is a comparison of the elements of the scattering matrix of the lined section of the test duct predicted by the present Q1D model with measurements.

2. Theoretical considerations

2.1. Quasi-one-dimensional unsteady flow equations

Consider a uniform straight duct and let the geometry of the duct be defined in an orthogonal Cartesian frame with axes x_1, x_2, x_3, or the parallel central axes x, y, z, where $x || x_1$ denotes the duct axis. The geometry is shown schematically in Fig. 1 for a rectangular duct, however, the subsequent analysis is applicable for any section shape. The unit vectors in positive directions of x_1, x_2, x_3 axes are denoted by $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$, respectively, and $\nabla = \mathbf{e}_1 \partial/\partial x_1 + \mathbf{e}_2 \partial/\partial x_2 + \mathbf{e}_3 \partial/\partial x_3$ denotes the gradient operator. The pressure, particle velocity and density of the fluid contained in the duct are denoted by p, $\mathbf{v}$ and ρ, respectively. Unsteady inviscid fluid flow in a source-free region of the duct is governed by the basic continuity and momentum equations [12].

$$\frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho + \rho \nabla \cdot \mathbf{v} = 0 \tag{1}$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) + \nabla p = 0 \tag{2}$$

respectively, where t denotes the time. The convected wave equation and the Pridmore-Brown equation that are relevant to the present study are derived from the linearized forms of these equations under the assumptions that the mean density and pressure are constant and the mean flow velocity profile is axially uniform [5]. Here, these wave equations are not required, as we proceed with the integral forms of Eqs. (1) and (2).