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The acoustic Green's function for swirling flow with variable entropy in a lined duct

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ABSTRACT

This paper extends previous work by the authors (*Journal of Sound and Vibration*, 395:294-316,2017) on the acoustic field inside an annular duct with acoustic lining carrying mean axial and swirling flow so as to allow for non-uniform mean entropy, as would be found for instance in the turbine stage of an aeroengine. The main aim of this paper is to understand the effect of a non-uniform entropy on both the eigenmodes of the flow and the Green's function, which will allow noise prediction once we have identified acoustic sources. We first derive a new acoustic analogy in isentropic swirling flow, which allows us to derive the equation the tailored Green's function satisfies. The eigenmodes are split into two distinct families, acoustic and hydrodynamic modes, and are computed using different analytical methods; in the limit of high reduced frequency using the WKB method for the acoustic modes; and by considering a Frobenius expansion for the hydrodynamic modes. These are then compared with numerical results, with excellent agreement for all eigenmodes. The Green's function is also calculating analytically using the realistic limit of high reduced frequency, again with excellent agreement compared to numerical calculations. We see that for both the eigenmodes and Green's function the effect of non-uniform mean entropy is significant.

1. Introduction

In light of increasing environmental concerns, operational targets and airport expansion it is vital to understand and model sound levels produced by aircraft engines accurately. One key noise source of the aircraft engine comes from the rotor and its interaction with the stator, for which understanding the propagation of acoustic waves through swirling, isentropic flow in an annular duct will be crucial and which we focus on in this paper. This analysis will also be useful in the turbine and compressor regions of the aeroengine, where the mean entropy is often non-uniform.

To calculate the propagation of noise in an aeroengine an acoustic analogy is often required. Lighthill [21] was the first to derive an acoustic analogy, famously rearranging the Navier–Stokes equations into a single equation, with the left-hand side the wave operator in a stationary fluid acting on the density perturbation. The right-hand side of the analogy consists of the remaining terms (including non-linear and viscous effects), and is thought of as the sound source. If the right-hand side is known, then the solution of Lighthill's analogy is given by a convolution of the source terms and the free space Green's function of the wave operator. Lighthill's analogy has been extended in numerous ways; Curle [8] and Ffowcs Williams and

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Hawkings [12] considered moving surfaces, while Goldstein [13] and Morfey and Wright [28] considered different dependent variables on the left-hand side. Often, the fluid is not stationary, and has a radially varying base flow, which requires a new acoustic analogy. Lilley [22] extended the acoustic analogy to flows with a unidirectional base shear flow, and the left-hand side of his operator is often approximated by the linear Pridmore-Brown equation [33] acting on the logarithm of the pressure perturbation. Finally, Posson and Peake [32] considered a swirling base flow in a duct and derived a sixth order linear operator acting on the pressure perturbation. For all of these acoustic analogies it will be necessary to find the Green's function of the left-hand side, and the complexity of this Green's function increases with the complexity of the base flow.

Once we have calculated the Green's function, we can predict the noise generated for simple source distributions, as performed by Posson and Peake [32] and Masson et al. [23] for swirling flow. Alternatively, the Green's function has applications in many other noise prediction methods such as beamforming and the two-microphone method. Beamforming is now one of the major processing tools to analyse data microphone array data in aeroengine noise tests, and the Green's function is used to infer information about the noise sources from the measurements. Although much recent progress has been made, such as by Sijtsma [35], the most state-of the art base flow currently being used is a piecewise constant unidirectional shear flow ([36]). Another potential application for the Green's function in swirling flow is the two-microphone method, which uses the Green's function to determine how noise propagates between two microphone measurements. It is currently being worked on by Joseph et al. [18], Chen and Joseph [4], but at present is restricted to a uniform axial base flow.

To evaluate the Green's function, we need to firstly calculate the eigenmodes of the linearised Euler equations about a swirling, isentropic base flow. These eigenmodes can be split into two distinct families. The acoustic modes are pressure-driven, while the hydrodynamic (or nearly convected) modes contain most of the unsteady vorticity of the flow. There is also the critical layer, which is a singularity of the system of linearised Euler equations. We can get rid of this artefact by including higher order terms or viscosity, but most of the time the effect of the critical layer is neglected since it is assumed to be small ([32]). Once we have calculated the eigenmodes, we find the Green's function by evaluating the residues of the acoustic eigenmodes, and neglecting the contributions from the hydrodynamic modes and critical layers.

The hydrodynamic modes were first shown to exist by Kerrebrock [19]. It was shown that there are infinitely many of them when we consider swirling mean flow by Golubev and Atassi [14], who were among the first to study the asymptotic behaviour of these modes. A further numerical study of the modes was performed by Nijboer [30]. The first full asymptotic treatment of the hydrodynamic modes was done by Heaton and Peake [17], who showed three possible asymptotic regimes of the hydrodynamic modes, depending on the flow parameters. It was shown that the modes could accumulate either exponentially or algebraically, with the latter splitting into two cases, on the real line and in the complex plane. Heaton and Peake's work addressed several issues with earlier work such as the use of a thin duct assumption in Golubev and Atassi [14].

There have been numerous studies about the acoustic eigenmodes. In Cooper and Peake [6], Heaton and Peake [16] the acoustic eigenmodes and eigenfunctions for swirling flow in a hard-walled infinite duct were calculated asymptotically, using the WKB method. In both papers they show we can get turning points in the WKB method, corresponding physically to caustics, Cooper and Peake [6, Figure 10]. In Vilenski and Rienstra [39,38] a lined infinite duct is considered, but for zero swirl. They seek the acoustic eigenmodes and eigenfunctions of the resulting Pridmore–Brown ([33]) differential equation, and compare their asymptotic method with some numerical results. In Posson and Peake [32] the eigenmodes and Green's functions were calculated numerically for swirling flow with constant entropy in a hard-walled infinite duct. In Posson and Peake [31] the results were extended to an infinite duct with acoustic lining. In Mathews and Peake [26] the eigenmodes and Green's functions were calculated analytically for swirling flow and compared to the numerical results, showing excellent agreement, even for the cases of semi-realistic swirling flow in a lined duct.

Relatively little work has been carried out on the effect of entropy on the eigenmodes and Green's function, although Tam and Auriault [37] considered a isentropic flow. They calculate eigenmodes and a Green's function in the case of simple swirling flow in an infinite hard-walled duct, and their choice of base flow density ensured the entropy of the base flow varied. In Cooper [5] the effect of entropy is considered on the propagation of the pressure field in a slowly varying duct, with most of the analysis only concerning the first cut-on eigenmode and associated eigenfunction.

1.1. *Organisation*

This paper is laid out as follows. In Section 2 we derive a new acoustic analogy, based on the work from Posson and Peake [32]. In Posson and Peake [32] the base flow used in the acoustic analogy is homentropic, while we allow the entropy to vary radially in the base flow. In Section 3 we revisit the asymptotic and numerical methods for finding the acoustic eigenmodes and Green's function in swirling, isentropic flow, with the methods based upon Mathews and Peake [26]. In Section 4 we consider the effect of entropy on the acoustic eigenmodes, and show that there are three main features due to the variable entropy. In Section 5 we consider the effect of entropy on the hydrodynamic modes, both numerically and asymptotically. We extend the work from Heaton and Peake [17] to an isentropic flow, and find there are still three accumulation regimes. Finally, in Section 6 we consider the effect of entropy on the Green's function, which is calculated by evaluating the residue at each of the acoustic modes.

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