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An adjoint method of sensitivity analysis for residual vibrations of structures subject to impacts

Kun Yan [a], Gengdong Cheng [b,*]

[a] School of Chemical Machinery and Safety, Dalian University of Technology, Dalian, Liaoning, 116024, PR China
[b] State Key Laboratory of Structural Analysis for Industrial Equipment, Faculty of Vehicle Engineering and Mechanics, Dalian University of Technology, Dalian, 116023, PR China

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For structures subject to impact loads, the residual vibration reduction is more and more important as the machines become faster and lighter. An efficient sensitivity analysis of residual vibration with respect to structural or operational parameters is indispensable for using a gradient based optimization algorithm, which reduces the residual vibration in either active or passive way. In this paper, an integrated quadratic performance index is used as the measure of the residual vibration, since it globally measures the residual vibration response and its calculation can be simplified greatly with Lyapunov equation. Several sensitivity analysis approaches for performance index were developed based on the assumption that the initial excitations of residual vibration were given and independent of structural design. Since the resulting excitations by the impact load often depend on structural design, this paper aims to propose a new efficient sensitivity analysis method for residual vibration of structures subject to impacts to consider the dependence. The new method is developed by combining two existing methods and using adjoint variable approach. Three numerical examples are carried out and demonstrate the accuracy of the proposed method. The numerical results show that the dependence of initial excitations on structural design variables may strongly affects the accuracy of sensitivities.

1. Introduction

Many engineering structures in their service life are subject to impact loads. The impact load may result in propagating wave or excessive long-standing structural vibration or even severe damage depending on the magnitude and duration of the impact load. The structural vibration responses after impacts are known as the residual vibration [1–3]. Fig. 1 shows the displacement response of a simple one degree of freedom system subject to an impact load. The dotted and solid curves are the structural responses of forced vibration and residual vibration, respectively. For many precision mechanical systems, excessive structural residual vibration is of important concern [4–6], such as the residual vibration of robot arm after knocking an obstacle is harmful to its performance [6].

Many active or passive approaches have been proposed for structural residual vibration reduction [7–9]. These methods can roughly be broken into two categories: 1) passive control such as hardware design and command shaping, 2) active

* Corresponding author.
E-mail address: chenggd@dlut.edu.cn (G. Cheng).

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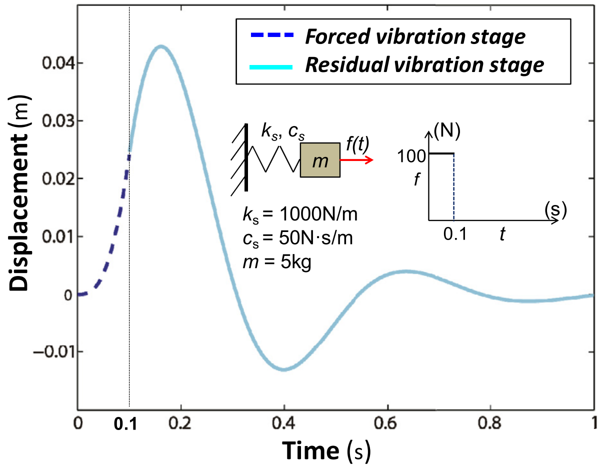

Fig. 1. Vibration response of a structure subject to impact.

feedback control. Here, passive control of structural residual vibration is considered. These reduction approaches of residual vibration often show better performance by using sensitivity analysis [10,11].

The sensitivities of residual vibration with respect to structural parameters reflect the structural response to changes of structure model or inputs [12–14]. In general, there are three approaches to sensitivity analysis: the finite difference method, the direct differentiation method and the adjoint method [15–17]. The finite difference method is a direct and simple approach, but suffers from computational inefficiency. Therefore, the direct differentiation method (DDM) or the adjoint variable method (AVM) is generally preferred despite their relative complexity [18]. The efficiency of these methods depends on the number of design variables and the number of active constraints [19]. When the number of design variable is large, e.g., in topology optimization problems, then AVM is preferred [20–24].

For structural dynamic problem, the structural response is time dependent and a global measure is often adopted to enhance the efficiency of numerical treatment in structural optimization. Consider a structure is to be designed and subject to impact. Its dynamic response is governed by Eq. (1a).

$$\mathbf{M}(\mathbf{d})\ddot{\mathbf{u}}(t) + \mathbf{C}(\mathbf{d})\dot{\mathbf{u}}(t) + \mathbf{K}(\mathbf{d})\mathbf{u}(t) = \mathbf{f}(t) \tag{1a}$$

where, t is time variable and starts from 0, $\mathbf{M}(N \times N)$ is the mass matrix, $\mathbf{C}(N \times N)$ is the damping matrix, $\mathbf{K}(N \times N)$ is the stiffness matrix, $\mathbf{d}$ is the structural design variable vector and $\mathbf{u}(N \times 1)$, $\dot{\mathbf{u}}(N \times 1)$, $\ddot{\mathbf{u}}(N \times 1)$ are the nodal displacement, velocity and acceleration vector, they all depend on the design variable **d.** N is the number of structural degree of freedoms. $\mathbf{f}(t)$ is the nodal impact load vector, and can be expressed as

$$\mathbf{f}(t) = f(t)\mathbf{f}_0 \quad f(t)\begin{cases} \neq 0 & 0 < t \leq T_\mathrm{d} \\ = 0 & t > T_\mathrm{d} \end{cases} f(t) \begin{cases} \neq 0 & 0 < t \leq T_\mathrm{d} \\ = 0 & t > T_\mathrm{d} \end{cases} \tag{1b}$$

where, T_d is the duration of impact. Assume that the structure is motionless at initial instance

$$\mathbf{u}(0) = 0 \quad \dot{\mathbf{u}}(0) = 0 \tag{1c}$$

the structural vibration $\mathbf{u}(t)$ is obtained by transient analysis of Eqs. (1a)~(1c) and consists of forced vibration ($0 \leq t \leq T_\mathrm{d}$) and residual vibration ($t \geq T_\mathrm{d}$).

For reduction of residual vibration response of structures subject to impact, a measure J in Eq. (2) is introduced, which is an integrated quadratic performance index and globally measures structural residual vibration [25,26].

$$J = \int_{T_\mathrm{d}}^{\infty} (\dot{\mathbf{u}}(t)^\mathrm{T}\mathbf{Q}_{\dot{\mathbf{u}}}\dot{\mathbf{u}}(t) + \mathbf{u}(t)^\mathrm{T}\mathbf{Q}_{\mathbf{u}}\mathbf{u}(t))\mathrm{d}t \tag{2}$$

where, $\mathbf{Q}_{\dot{\mathbf{u}}}$ and $\mathbf{Q}_{\mathbf{u}}$ are positive or semi-positive definite symmetric weighting matrices. The direct calculation of the quadratic performance index in Eq. (2) is time consuming, because it needs to perform transient dynamic analysis. By Lyapunov equation, the calculation of Eq. (2) can be greatly simplified into computing matrix quadratic forms without transient analysis when every degree of freedom is damped. With the Lyapunov equation, sensitivity analysis methods for the quadratic performance index have been proposed [27–32]. Wang et al. [27] applied the Lyapunov equation to solve the transient response optimization problem of linear vibrating systems excited by initial conditions and proposed a sensitivity analysis method