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Characterization of low-frequency acoustic wave propagation through a periodic corrugated waveguide

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A B S T R A C T

In this paper, a periodic corrugated waveguide structure is proposed, and its unit-cell is analyzed by the wave finite element method. In low-frequency range, the unit-cell is treated as an equivalent fluid through a homogenization process, and the equivalent acoustic parameters are obtained, which are validated by finite structure simulations and experiments. The proposed structure is shown to add tortuosity to the waveguide, hence higher equivalent fluid density is achieved, while the system elastic modulus remains unchanged. As a result, the equivalent speed of sound is smaller than normal air. The application of such change of speed of sound is demonstrated in the classic quarter-wavelength resonator based on the corrugated waveguide, which gives a lower resonance frequency with the same side branch length. When the waveguide is filled with porous materials, the added tortuosity enhances the broadband, low-frequency sound absorption by increasing the equivalent mass without bringing in excess damping, the latter being partly responsible for the poor performance of usual porous materials in the low-frequency region. Therefore, the proposed structure provides another dimension for the design and optimization of porous sound absorption materials.

1. Introduction

The problem of acoustic wave propagation in the periodic waveguide has attracted researchers' attention in recent years, as it has unique physical characteristics. A periodic waveguide is often composed of a waveguide with periodic wall undulations or periodic scatterers. Some earlier researchers investigated the wave propagation characteristics in a waveguide with sinusoidal walls. They found that the wavy walls may produce disturbance to the incident wave in the form of two coupling modes [1–4]. Bostrom investigated this phenomenon further by calculating the dispersion curves for the axial wavenumber of a duct with periodically varying cross-section, and obtained the expected passband and stopband, which can be explained by Bragg resonance for general periodic structures [5]. Munday et al. studied the stopband of a diameter-modulated waveguide theoretically and experimentally, and found that the defects in the perfect periodicity may result in a narrow transmission band in the stopband [6]. Recently, Tao et al. studied the non-Bragg resonance in the waveguide whose walls have periodic undulations, which can form another type of stopband [7,8] and a narrow transmission band in the Bragg

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stopband [9]. These researches mainly focus on the acoustic wave propagation characteristics in the stopband, which is in relatively high-frequency range.

Sugimoto and Horioka explored the dispersion characteristics of the acoustic wave in a straight waveguide with periodic side branch Helmholtz resonators, and found stopbands caused by both Bragg resonance and side branch resonance [10]. Such type of periodic waveguide may exhibit interesting characteristics around the side branch resonance frequency, like the slow wave propagation [11,12] and negative equivalent parameters [13]. However, these characteristics exist only in a narrow frequency band around the resonance frequency, which cannot be generalized to broader band.

Bai and Keller investigated the characteristics of the acoustic wave propagation in a waveguide with periodic rigid spheres embedded inside [14]. They also found the passband and the stopband. Besides, they found the decreased phase velocity in the first passband. However, they did not provide physical interpretations.

In this paper, a periodic corrugated waveguide (PCWG) structure is proposed, and its characteristics in low-frequency range are analyzed with equivalent acoustic parameters obtained through a homogenization process. These acoustic parameters, such as equivalent speed of sound and density, are analyzed deeply with physical interpretations. Besides, the proposed PCWG structure provides a new dimension for the design and optimization of porous materials, which can enhance sound absorption properties of porous materials in low-frequency range.

For the calculation method of a periodic waveguide, a perturbation expansion solution is available when the undulation of walls [15] or the diameter of the interior scatterer [14] is relatively small. In 1994, Bradley proved that the wave propagating in a periodic waveguide is a Bloch type wave, as long as all higher-order modes are evanescent [16]. Therefore, when the geometry of the waveguide is simple enough, a standard eigenvalue problem can be formed with the help of the Bloch-type periodic condition and transfer matrix method [10]. In addition, in the low-frequency range, two variants of the transfer matrix method, the transfer reflection coefficient method [6] and transfer impedance method [13] can be applied, both achieving the periodic setting by recursion instead of using the Bloch periodic condition directly. The methods mentioned above are analytical or semi-analytical, which can only deal with simple geometries. The development of finite element (FE) method makes it possible to solve the problem with complex geometries [17]. In 1973, Mead proposed a generalized theory to deal with harmonic wave propagation in periodic structures, which allows the coupling of any number of coordinates between adjacent units [18]. This method could be implemented with FE method to deal with non-uniform periodic structures, which was conducted by Orris and Petyt in the following year [19]. In 2005, Mace et al. presented another eigenvalue problem formulation to calculate waveguide problems with the derived stiffness, damping and mass matrixes in classic FE method, which was referred to as wave finite element (WFE) method thereafter [20]. The eigenvalue corresponds to the propagation constant and eigenvector corresponds to the eigenmode shape. One of the major advantages of the WFE method is that it can make use of any mature commercial code, instead of building the FE matrix from scratch. Since then, the WFE method has been employed in various fields for the analysis of free and forced vibration problems, e.g. beams [21,22], pipes [23–25], plates [26,27], tires [28], shells [29] and periodic structures with complicated unit-cells [30]. In this paper, the proposed PCWG structure is modelled with WFE method to form a standard eigenvalue problem.

This paper is arranged as follows. Section 2 presents the eigenvalue problem formulation of the proposed PCWG structure with the WFE method. In Section 3, the homogenization results of equivalent parameters are presented and analyzed. In Section 4, the effect of the PCWG on enhancing low-frequency sound absorption for porous materials is discussed. The conclusions are drawn in Section 5.

2. Methodology

The geometries of the periodic corrugated waveguide (PCWG) and its unit-cell are shown in Fig. 1, as well as the coordinate system, and the wave travels in the direction of x. The length of the unit-cell in the x direction is d, and the height of the unit-cell is h_0. In the unit-cell, the interior thick lines are the interior sound-hard walls, which means that normal acoustic particle velocity vanishes on this boundary. Boundaries AD and BC are rigid boundaries of the waveguide. Periodic boundary condition is applied on boundaries AB and CD. The dot-dash lines are the axes of symmetry, so the geometry of the unit-cell is determined by the geometric parameters of h_0, d, L_1, L_Δ, L_2, h_1 and h_2, as shown in the lower part of Fig. 1.

The calculation is conducted in the frequency domain, so the time-dependent factor $e^{i\omega t}$ is omitted, where $i=\sqrt{-1}$, ω is the angular frequency and t represents time. The governing equation for the unit-cell is the linear homogeneous Helmholtz equation

$$\nabla^2 p + k^2 p = 0, \tag{1}$$

where p is the acoustic pressure, k is the wavenumber of the material inside the unit-cell. The particle velocity $\boldsymbol{v}$ is calculated as:

$$\boldsymbol{v} = -\frac{1}{i\omega\rho}\nabla p, \tag{2}$$

where ρ is the density of the material inside the unit-cell.

The upper and lower boundaries (AD and BC) are hard walls where the normal particle velocity vanishes: