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Dynamics of unforced and vertically forced rocking elliptical and semi-elliptical disks

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ABSTRACT

This paper presents the results of an investigation on the dynamics of unforced and vertically forced rocking elliptical and semi-elliptical disks. The full equation of motion for both rocking disks is derived from first principles. For unforced behavior, Lamb's method is used to derive the linear natural frequency of both disks, and harmonic balance is used to determine their amplitude-dependent rocking frequencies. A stability analysis then reveals that the equilibria and stability of the two disks are considerably different, as the semi-elliptical disk has a supercritical pitchfork bifurcation that enables it to exhibit bistable rocking behavior. Experimental studies were conducted to verify the trends. For vertically forced behavior, numerical investigations show the disk's responses to forward and reverse frequency sweeps. Three modes of periodicity were observed for the steady state behavior. Experiments were performed to verify the frequency responses and the presence of the three rocking modes. Comparisons between the experiments and numerical investigations show good agreement.

1. Introduction

The dynamic behavior of rocking disks is a classical problem that has been studied by scientists, engineers, and mathematicians. The problem can be investigated from a variety of perspectives, and is a useful tool for providing insights into the dynamics of other, more complicated systems. Two of the most well-known cases of rocking disk problems are Euler's disk and the rattleback, which both exhibit unusual dynamical behavior. Euler's disk refers to a circular disk of uniform density that rolls on a flat surface, and the system initially received interest in the scientific community because the speed of the disk's rolling paradoxically increases as energy is dissipated from the system. Moffat conducted the initial study on the dynamical behavior of Euler's disk, and he hypothesized that the paradoxical nature of the system was a result of viscous dissipation caused by the sheared air between the disk and the ground [1]. Moffatt was able to discover a finite-time singularity that explained the phenomenon. This seminal work sparked several additional studies which questioned whether the primary dissipative elements in the model should be viscous air effects or sliding friction at the contact point [2–4], and current opinions tend to support the sliding friction hypothesis [5]. Several studies have been conducted in recent years in an attempt to match experimental results with theory [5,6].

The rattleback is a perplexing device that was first discovered at ancient Celtic archaeological sites in the 19th century. In fact, rattlebacks were initially referred to as celts due to the origin of their discovery. Rattlebacks come in a variety of shapes and sizes, but can typically be described as mechanical tops with a smoothly curved surface and a preferred direction of spin caused by geometric asymmetry. If the rattleback is spun in its stable direction, it simply maintains spin in that direction until

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the motion is damped out. However, if the rattleback is spun in its unstable direction, it begins to “rattle” and then reverses the direction of its spin to the stable direction. Some rattlebacks can be designed so that they are unstable in both directions and can therefore exhibit multiple spin reversals. The dynamics of rattleback motion were first derived in the late 1890s by Walker [7]. There was a renewed interest in rattleback dynamics in the 1980s, and several interesting studies were published. Lindberg and Longman verified Walker’s model through numerical simulation [8], Kane and Levinson modeled the effects of dissipation on rattleback dynamics [9], Bondi classified different types of rattlebacks based on their behavior [10], and Garcia and Hubbard validated their rattleback model with experimental results [11].

This paper will focus on another class of rocking disk problem that was initially studied by Lamb in the early 1900s and has received somewhat less attention than Euler’s disk or the rattleback. In his initial work, Lamb derived the linear natural frequency of an arbitrary disk with uniform density and a symmetric cross section [12]. This linear natural frequency is the rocking frequency of the disk for small angles of oscillation. Satterly used Lamb’s equation to derive the linear natural frequency of an elliptical disk [13], and Balachandran derived the linear natural frequency of a semicircular disk by deriving the system’s equation of motion [14]. A recent study used rapid prototyping technology to experimentally validate Lamb’s method for calculating the linear natural frequency of rocking semicircular and parabolic disks [15]. These studies focused on linear analysis, and they largely neglected the effects of nonlinearity on the dynamic behavior of the disk. This paper builds upon these previous studies and shows how nonlinearity significantly influences the dynamics of rocking elliptical and semi-elliptical disks through bifurcations and amplitude-dependent phenomena.

Another perspective that has not yet been fully investigated is the behavior of rocking disks under vertical excitations. Mazzoleni analyzed the stability and bifurcations for the unforced behavior of rocking disks, but did not fully derive the equations of motion or investigate forced behaviors [16]. Due to the geometry of an elliptical or semi-elliptical rocking disk, a small vertical excitation can be transferred into magnified horizontal and rotational movements for the purpose of energy harvesting. This transmission of translational excitation into rotational motion could have a wide range of applications. For example, pendulums are widely used for energy harvesting but few of them are vertically driven due to gravity in the vertical direction. Mann’s bistable magnetic pendulum [17], Toh’s marine energy harvester [18] and Wang’s weighted-pendulum-type electromagnetic generator [19] use either horizontal or rotational excitation to drive pendulums. Fortunately, if a pendulum is pinned to a rocking disk, the vertically excited rocking motion could easily drive the pendulum. Mounting a pendulum to a ship is one of the simplest methods to take advantage of a ship’s rocking motion to harvest potential energy from a wave [20]. Therefore, it is worthwhile to investigate the response of a driven rocking disk and find ways to alter its frequency response.

The remainder of this paper is organized as follows. Section 2 derives the equation of motion for the forced rocking elliptical and semi-elliptical disks. Section 3 investigates the unforced behavior for both rocking elliptical and semi-elliptical disks. It includes four subsections that: 1) analyze the linear natural frequencies of disks using Lamb’s method, 2) derive an approximate analytical solution for the disk’s amplitude-dependent rocking frequency, 3) analyze the equilibria and stability of the rocking disks using phase plane analysis and the Lagrange-Dirichlet theorem [21], and 4) perform experimental validation of the linear natural frequencies and bifurcation trends. Section 4 investigates the vertically forced behavior of a monostable semi-elliptical disk. In this section, the rocking disk’s responses to forward and reverse frequency sweeps are shown from simulations and three modes of periodicity for steady states were observed. Experiments were performed to verify the frequency responses and the presence of the three rocking modes and two cut-off frequencies.

2. Equation of motion

This section derives the equation of motion for rocking elliptical and semi-elliptical disks that are forced by vertical excitations. The geometries for these disks that will be analyzed in this paper are depicted in Fig. 1. The surfaces of the disks are defined by $x^2/a^2 + y^2/b^2 = 1$, where a and b are the major and minor radii of the disks, respectively. When deriving the equation of motion, the case of the semi-elliptical disk was used, since the governing equation for the elliptical disk can be obtained from the governing equation for the semi-elliptical disk through two parameter substitutions. Fig. 2 shows the upright and displaced position of a rocking semi-elliptical disk with a vertical excitation applied to the base. It is assumed that the disk rolls without slip on the base.

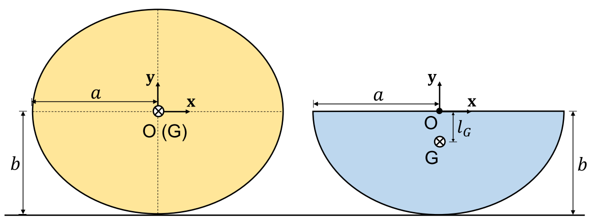

Fig. 1. Geometries of the elliptical and semi-elliptical disks with surfaces defined by $x^2/a^2 + y^2/b^2 = 1$. For the elliptical disk, the geometric center O and the center of mass G coincide.