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Time reversal invariance for a nonlinear scatterer exhibiting contact acoustic nonlinearity

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The time reversal invariance of an ultrasonic plane wave interacting with a contact interface characterized by a unilateral contact law is investigated analytically and numerically. It is shown analytically that despite the contact nonlinearity, the re-emission of a time reversed version of the reflected and transmitted waves can perfectly recover the original pulse shape, thereby demonstrating time reversal invariance for this type of contact acoustic nonlinearity. With the aid of finite element modelling, the time-reversal analysis is extended to finite-size nonlinear scatterers such as closed cracks. The results show that time reversal invariance holds provided that all the additional frequencies generated during the forward propagation, such as higher harmonics, sub-harmonics and zero-frequency component, are fully included in the retro-propagation. If the scattered waves are frequency filtered during receiving or transmitting, such as through the use of narrowband transducers, the recombination of the time-reversed waves will not exactly recover the original incident wave. This discrepancy due to incomplete time invariance can be exploited as a new method for characterizing damage by defining damage indices that quantify the departure from time reversal invariance. The sensitivity of these damage indices for various crack lengths and contact stress levels is investigated computationally, indicating some advantages of this narrowband approach relative to the more conventional measurement of higher harmonic amplitude, which requires broadband transducers.

1. Introduction

The application of nonlinear acoustics for nondestructive evaluation and structural health monitoring has attracted considerable research interest over the past two decades, driven by the prospect that various forms of structural damage may induce a nonlinear response that could lead to earlier damage detection, without the need for baseline data, than would be possible through conventional linear ultrasonics [1–3]. It is pertinent to distinguish between cases where the structural damage (and hence the source of nonlinearity) is uniformly distributed throughout the structure, so that the response can be adequately modelled by an appropriate nonlinear constitutive equation for the material [4–6], and cases where the damage is

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localized, with the surrounding material behaving linearly. The present work is concerned with the latter case. In practice, two important forms of localized damage are fatigue cracks in structural alloys and delaminations in composite laminates. For both cases, the nonlinear response can generally be attributed to contact acoustic nonlinearity (CAN) [7–15], which induces the generation of new frequency components such as higher harmonics, sub-harmonics, and zero frequency (DC) response. The mechanisms involved in CAN include clapping between the contacting surfaces [7,9], as well as dissipative mechanisms due to frictional sliding [8,12]. Theoretical models of varying levels of sophistication have been proposed for all of these mechanisms, as comprehensively reviewed in Ref. [14]. We note in particular the recent work on the vibrational response of beams with breathing cracks [15–18], of plates with delaminations [19,20], as well as wave scattering by cracks and delaminations [1,21–25], as examples of the current state of the art for CAN modelling.

The principle of time-reversal invariance in non-dissipative media relies on the fact that the governing equations of motion only involve a second-order time derivative, which is invariant with respect to time reversal, whereas dissipative mechanisms introduce odd-order derivatives that break the time-reversal symmetry [26–28]. A full-field time reversal requires that the forward propagating field should be recorded on a closed surface in 3D, or a closed curve in 2D, prior to time reversal of the recorded signals. The re-emission then produces a backward propagating field that exhibits spatial and temporal focusing at the locations of point-like sources, or scatterers. Such a closed surface in 3D, or closed curve in 2D, is referred to as a time-reversal cavity [26–28]. In most applications of time reversed acoustics to date, it is the spatial retro-focusing that is of primary interest [26–28]. The practical implementation of a 3D time reversal cavity is difficult, and this has prompted the use of time reversal mirrors instead [27,28]. These mirrors have a finite angular aperture, which is reflected in a broadening of the focal spot, i.e. a loss of resolution, which may nevertheless be acceptable for the intended applications [27]. For 2D applications, however, such as Lamb wave structural health monitoring, it is relatively easy to implement sensor arrays on a closed curve surrounding a region of interest [29–42]. Indeed such arrays have proved effective for imaging extended (i.e. non-point-like) structural damage in plate-like structures [43–46]. The relevant imaging algorithms, as well as strategies for minimizing the required number of sensors, have been discussed in Refs. [47,48].

The present work aims to investigate whether, from a fundamental perspective, CAN will necessarily lead to a breakdown of time reversal (TR) invariance, as it has been implicitly assumed in previous work [49,50]. This investigation is therefore complementary to the recent demonstration by Tanter et al. [51] that TR invariance still holds in a nonlinear acoustic medium, provided that the time reversal is implemented within a propagation distance that is shorter than the shock-formation distance [52]. In that case, the wave form is progressively recovered during the back propagation, and the distortions accumulated during the forward propagation are cancelled. By contrast, with CAN the source of nonlinearity is localized at a contact interface, whereas the propagation medium is linear. In a recent study [53], the present authors showed that TR invariance holds for a 1D computational model of CAN involving only the reflection at a fixed contact interface, as in the experimental set up of Solodov [7]. This theoretical insight had not previously been recognized. The present work extends this result in two important ways.

First, the 1D model is extended by involving both transmission and reflection. Although it may be considered plausible that TR invariance should still apply in this more general case, this plausibility does not constitute a proof. An analytical proof is provided here in Section 3, by contrast to the computational verification that was presented in Ref. [53] for the case of reflection only. This analytical proof is based on a model of CAN first derived by Richardson [54]. The key aspects of this model are summarized here in Section 2.1 as a pre-requisite for the proof presented in Section 2.2. Secondly, a 2D (plane strain) model of CAN is investigated computationally in Sections 3 and 4, using a geometrical configuration that allows for a convenient implementation of the closed-cavity requirement for full-field time reversal. For this numerical verification of TR invariance, it is also necessary to use a sufficiently broad frequency bandwidth, to ensure an adequate capture of the higher harmonics generated by the nonlinear interaction. Section 5 investigates what would happen if a narrowband capture and re-emission is implemented instead, to simulate what might be observed in practice when using narrowband resonant transducers. TR invariance is then lost, but this can be attributed to an inadequate implementation of the TR protocol, rather than being a fundamental property of the present model of CAN. Various measures for quantifying this breakdown of TR invariance are formulated in Section 5, and their performance as potential damage indices is assessed. The breakdown of TR invariance when using only one pair of active sensors, which can also be regarded as an inadequate implementation of the TR protocol, is also discussed briefly. Finally, Section 6 provides concluding remarks.

2. Theoretical demonstration of TR invariance for a unilateral contact model

The system considered here consists of an unbounded planar interface separating two identical semi-infinite elastic media. The medium on either side of the interface is assumed to behave linearly. An initial state of hydrostatic stress is assumed to prevail, i.e. a negative contact stress σ_0 exists across the interface. As a result the interface cannot transmit a tensile stress exceeding $|\sigma_0|$, and any wave with a maximum tensile stress less than $|\sigma_0|$ can propagate unhindered across the interface. When the incident tensile stress exceeds $|\sigma_0|$, the interface opens, giving rise to distorted reflected and transmitted pulses. An analytical solution for the forward propagation problem is presented below (Section 2.1), adapting the approach of Richardson [54], with minor changes in notation. Once the forward solution is established, the backward propagation problem is analysed in a similar approach to demonstrate the TR invariance for this model of CAN (Section 2.2).