Journal of Sound and Vibration 417 (2018) 359–375

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

An efficient analytical model for baffled, multi-celled membrane-type acoustic metamaterial panels

F. Langfeldt [a,*], W. Gleine [a], O. von Estorff [b]

[a] Department of Automotive and Aeronautical Engineering, Hamburg University of Applied Sciences, Berliner Tor 9, D–20099 Hamburg, Germany
[b] Institute of Modelling and Computation, Hamburg University of Technology, Denickestr. 17, D–21073 Hamburg, Germany

ARTICLE INFO

Article history:
Received 9 August 2017
Revised 4 December 2017
Accepted 8 December 2017
Available online XXX

Keywords:
Metamaterial
Membrane
Transmission loss
Diffuse
Baffled
Effective mass

ABSTRACT

A new analytical model for the oblique incidence sound transmission loss prediction of baffled panels with multiple subwavelength sized membrane-type acoustic metamaterial (MAM) unit cells is proposed. The model employs a novel approach via the concept of the effective surface mass density and approximates the unit cell vibrations in the form of piston-like displacements. This yields a coupled system of linear equations that can be solved efficiently using well-known solution procedures. A comparison with results from finite element model simulations for both normal and diffuse field incidence shows that the analytical model delivers accurate results as long as the edge length of the MAM unit cells is smaller than half the acoustic wavelength. The computation times for the analytical calculations are 100 times smaller than for the numerical simulations. In addition to that, the effect of flexible MAM unit cell edges compared to the fixed edges assumed in the analytical model is studied numerically. It is shown that the compliance of the edges has only a small impact on the transmission loss of the panel, except at very low frequencies in the stiffness-controlled regime. The proposed analytical model is applied to investigate the effect of variations of the membrane prestress, added mass, and mass eccentricity on the diffuse transmission loss of a MAM panel with 120 unit cells. Unlike most previous investigations of MAMs, these results provide a better understanding of the acoustic performance of MAMs under more realistic conditions. For example, it is shown that by varying these parameters deliberately in a checkerboard pattern, a new anti-resonance with large transmission loss values can be introduced. A random variation of these parameters, on the other hand, is shown to have only little influence on the diffuse transmission loss, as long as the standard deviation is not too large. For very large random variations, it is shown that the peak transmission loss value can be greatly diminished.

1. Introduction

More than 15 years ago, acoustic metamaterials have come up as promising technologies for the control of sound propagation [1–3]. Since Liu et al. [4] first proposed and demonstrated the unusual properties of locally resonant sonic materials, i.e. acoustic metamaterials that consist of multiple subwavelength sized resonant unit cells, many different types of acoustic metamaterials with a wide range of, to some extent exotic, applications have been developed. For example, acoustic metamaterials with negative values for the effective density [4–8] or effective bulk modulus [9–11] have been widely investigated. Furthermore, double-negative acoustic metamaterials with simultaneously negative values for the effective density and bulk modulus

* Corresponding author.
E-mail address: Felix.Langfeldt@haw-hamburg.de (F. Langfeldt).

https://doi.org/10.1016/j.jsv.2017.12.018

exhibiting unconventional wave phenomena (such as negative refraction and reversed doppler effects) have also been developed [12–17]. These designs enabled many different applications of acoustic metamaterials, for example in sound insulation [18–21], vibration damping [22], super- and hyperlensing [23–25], or acoustic cloaking [26–29].

Among the variety of acoustic metamaterial realizations, the so-called membrane-type acoustic metamaterials (MAMs) have drawn particular attention, especially in the field of noise control engineering [30]. As originally proposed by Yang et al. [5], MAMs are very flat and lightweight structures composed of a thin membrane layer with small attached masses. Due to anti-resonance mechanisms, MAMs exhibit narrow frequency bands at low frequencies (< 1000 Hz) with large transmission loss values which substantially exceed the corresponding mass-law value. This makes MAMs particularly useful for low-frequency noise control applications with strict limitations on the mass and construction space penalty of noise reduction means, such as aircraft and automobiles [31–33].

The MAM anti-resonances can be easily tuned to a desired frequency range by varying the properties of the membrane (e.g. the prestress) and masses (e.g. the weight) [5,34,35]. Further investigations have revealed that the sound insulation properties of MAMs can be further improved by stacking multiple MAM layers [18,36], using multiple masses in each unit cell [37], perforating the membrane [38], using active approaches [39,40], and employing multi-celled MAM arrays with purposefully detuned unit cells [41,42]. Most of the present investigations of the acoustic properties of MAMs (both experimental as well as analytical/numerical) have only considered small-scale samples under normal incidence of plane acoustic waves. However, such idealized conditions are rarely found in practice. Naify et al. [36] pointed out that the performance of MAMs could be compromised in a large-scale multi-celled arrangement because of the compliance of the support structure carrying the MAM unit cells and the mutual acoustic coupling between the unit cells. The experimental results by Varanasi et al. [43,44] for the diffuse incidence transmission loss of a large-scale multi-celled metamaterial structure with very similar properties to MAMs indicated that the acoustic properties of multi-celled metamaterial panels under diffuse sound field excitation can be considerably different from the performance of an equivalent small-sized sample under normal incidence. Langfeldt et al. [32], on the other hand, experimentally showed that a 1.2 m^2-sized MAM panel with 120 unit cells under diffuse field excitation exhibits a similar – though somewhat reduced – performance than in case of a single unit cell under normal excitation. More experimental investigations of large-scale MAM panels have not been published at the present time. Therefore, there is a need for further understanding of the sound transmission properties of large multi-celled MAM panels under non-uniform acoustic excitations.

In view of the benefits of using MAM panels with detuned unit cells for increasing the bandwidth of the panel [41,42], an efficient analytical model for predicting the sound transmission loss of such structures would significantly aid the design and optimization of metamaterial panels. Most current MAM models are restricted to individual unit cells and normal incidence [35,45–47]. Blevins [48] provided a strongly simplified analytical model for rapidly evaluating the sound transmission loss of multi-celled MAM panels. This model was applied to investigate the influence of the number of unit cells in an MAM array as well as the effect of using unit cells with different masses in a 2×2 array configuration. However, since this model is based upon the MAM unit cell theory of Zhang et al. [35], it does not take into account the bending stiffness of the added masses which is important to accurately predict the resonances and anti-resonances of MAMs [45–47]. Furthermore, Blevins' model neglects the mutual acoustic coupling of adjacent MAM unit cells which is expected to have a significant impact on the sound transmission of large multi-celled MAM panels [36].

In order to contribute to these investigations of multi-celled MAM panels, this paper provides a new analytical approach to accurately and efficiently estimate the sound transmission loss of baffled multi-celled MAM panels. In contrast to the MAM array model in Ref. [48], the present analytical model includes the accurate representation of the added mass stiffness as well as the mutual acoustic coupling between unit cells. Furthermore, the proposed analytical model allows the calculation of the diffuse field transmission loss of the MAM array, such that the performance of MAMs under more realistic sound excitation conditions, in contrast to the normal incidence considered in most of the previous MAM investigations, can be evaluated.

The paper is organized as follows: Section 2 provides the theoretical framework for the new MAM array model. This model is verified using finite element simulations in Section 3 under normal and diffuse incidence conditions. The computational efficiency of the analytical model is evaluated by comparing CPU times and memory consumptions with those of the simulations. At the end of Section 3, the impact of elastic MAM edges is discussed using numerical results. In Section 4, the results for the variation of different MAM array parameters are investigated using the analytical model. Finally, the conclusions of this paper are summarized in Section 5.

2. Analytical model

Fig. 1 shows the basic structure of a baffled MAM panel consisting of multiple rectangular unit cells. A Cartesian coordinate system (x, y, z) is defined in one corner of the panel with the x- and y-axes pointing along the two perpendicular edges of the panel. Thus, the panel and the rigid baffle surrounding it are located in the xy-plane and the z-axis is perpendicular to the panel surface. The size of the MAM unit cells is assumed to be equal, with l_x and l_y corresponding to the unit cell edge lengths in x- and y-direction, respectively. The number of unit cells in x- and y-direction are N_x and N_y, respectively. Thus, the total number of unit cells is given by $N_C = N_x \cdot N_y$ and the edge lengths of the whole panel are $L_x = N_x \cdot l_x$ and $L_y = N_y \cdot l_y$. Each unit cell within the panel can be uniquely identified by the index pair (p, q), with $p = 1, 2, \dots, N_x$, $q = 1, 2, \dots, N_y$, and $(1, 1)$ corresponding to the unit cell at the origin of the global coordinate system (see Fig. 1). The properties of every MAM unit cell (such as the location of the added mass(es)) are described in terms of local coordinate systems $(\widetilde{x}_{pq}, \widetilde{y}_{pq}, z)$ which are related to the global x- and