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Dynamics of a distributed drill string system: Characteristic parameters and stability maps

Ulf Jakob F. Aarsnes [a,b,*], Nathan van de Wouw [c,d,e]

[a] International Research Institute of Stavanger (IRIS), Oslo, Norway
[b] DrillWell - Drilling and Well Centre for Improved Recovery, Stavanger, Norway
[c] Eindhoven University of Technology, Department of Mechanical Engineering, Eindhoven, The Netherlands
[d] Department of Civil, Environmental & Geo-Engineering, University of Minnesota, Minneapolis, USA
[e] Delft Center for Systems and Control, Delft University of Technology, Delft, The Netherlands

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ABSTRACT

This paper involves the dynamic (stability) analysis of distributed drill-string systems. A minimal set of parameters characterizing the linearized, axial-torsional dynamics of a distributed drill string coupled through the bit-rock interaction is derived. This is found to correspond to five parameters for a simple drill string and eight parameters for a two-sectioned drill-string (e.g., corresponding to the pipe and collar sections of a drilling system). These dynamic characterizations are used to plot the *inverse gain margin* of the system, parametrized in the non-dimensional parameters, effectively creating a stability map covering the full range of realistic physical parameters. This analysis reveals a complex spectrum of dynamics not evident in stability analysis with lumped models, thus indicating the importance of analysis using distributed models. Moreover, it reveals trends concerning stability properties depending on key system parameters useful in the context of system and control design aiming at the mitigation of vibrations.

1. Introduction

The performance of rotary drilling systems used to drill boreholes in the earth is often limited by the occurrence of self-excited vibrations. Self-excited vibrations cause early fatigue of drill pipes and premature failure of bits and, therefore, should be avoided. Through a combination of damage to equipment, and increased downtime, drill string vibrations have been reported to account for 2–10% of well costs [1]. As fixed cutter bits (also known as PDC bits) are especially prone to self-excited vibrations, drilling systems employing these bits have seen significant scrutiny and attempts at mitigation of these vibrations.

To explain the occurrence of self-excited (axial and torsional) drill-string vibrations, a model of the relevant torsional and axial dynamics with an unstable equilibrium is needed. The implications of an equilibrium in this system being unstable is that a small initial perturbation to the system will grow over time and will, in severe cases, manifest itself as an observable oscillatory phenomena such as stick slip vibration or bit bouncing.

The cause of this instability in drilling systems with PDC bits have previously been explained by a rate-weakening effect[1] in the bit-rock interface law causing an instability in the torsional dynamics [2], and approaches to mitigating the oscillations by dealing with this instability directly started in the 90’s [3,4]. Such approaches, although having been reasonably successful [5,6],

* Corresponding author. International Research Institute of Stavanger (IRIS), Forskningsparken AS, Gaustadalléen 21, 0349 Oslo, Norway.
E-mail addresses: ujfa@iris.no (U.J.F. Aarsnes), n.v.d.wouw@tue.nl (N. van de Wouw).

[1] Essentially a Stribeck-like effect.

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Nomenclature

Independent variables

s Laplace variable
t Time in seconds
x Axial position in meters

Independent variables

w Force
τ Torque
v Axial velocity a delay differential equation which is in turn used to derive the minimal set of characteristic parameters that can be used to specify the linearized system dynamics. Based on the Nyquist criterion, the stability analysis is initiated in Section
ω Angular velocity

Laplace transformed states *The subscripts* $i \in \{0, b, p\}$ *denotes 'top-drive', 'bit', and 'pipe-collar interface', respectively*

W_i Weight
T_i Torque
V_i Axial velocity
Ω_i Angular velocity

Drill string properties *The superscripts* p, c *denotes the pipe and collar section respectively*

A Cross sectional area
E Young's modulus
J Polar moment of inertia
G Shear modulus
ρ Pipe density
k_a, k_t Axial/torsional viscous damping coefficient

Bit rock interaction (BRI) parameters

a Bit radius
ζ Cutting force inclination
ϵ Intrinsic specific rock energy
N Number of cutters

Operational parameters

$\overline{v}$ Imposed steady-state axial velocity
$\overline{\omega}$ Imposed steady-state angular velocity

Non-dimensional characteristic quantities, for $i \in \{a, t\}$ ***(axial,torsional)***

$\eta_i = -k_i^d k_i^L$ Pseudo reflection coefficient
$\overline{K}_i = t_* K_i$ Nominal loop gain
$\overline{\Omega} = 1/\overline{t}_N = \frac{t_* N}{2\pi}\overline{\omega}$ Steady-state angular velocity
$\overline{c} = \frac{c_a}{c_t} \approx 1.6$ Relative wave speeds
$\overline{\zeta}_i = \frac{\zeta_i^c}{\zeta_i^p}$ Relative collar to pipe impedance
$\eta_i^c = \frac{1-\overline{\zeta}_i}{1+\overline{\zeta}_i} e^{-k_i^c t_i^c}$ Collar section psuedo reflection coefficient
$\overline{t}_p = \frac{t_t^p}{t_*} = \frac{L^p}{L^p+L^c}$ Drill pipe travel time

Non-dimensional independent variables, with $t_* = t_t L/c_t$

$\overline{s} = s t_* = \mathrm{j}\varpi$ Dimensionless Laplace variable
ϖ Dimensionless frequency variable