Journal of Sound and Vibration 416 (2018) 192–212

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Experimental feedback linearisation of a vibrating system with a non-smooth nonlinearity

D. Lisitano [a], S. Jiffri [b], E. Bonisoli [c], J.E. Mottershead [b, *]

[a] Department of Management Production Engineering, Politecnico di Torino, Torino, 10129, Italy
[b] School of Engineering, University of Liverpool, Liverpool, L69 3GH, United Kingdom
[c] Department of Mechanical and Aerospace Engineering, Politecnico di Torino, Torino, 10129, Italy

ARTICLE INFO

Article history:
Received 13 July 2017
Received in revised form 22 November 2017
Accepted 24 November 2017

Keywords:
Non-smooth systems
Active control
Feedback linearisation

ABSTRACT

Input-output partial feedback linearisation is demonstrated experimentally for the first time on a system with non-smooth nonlinearity, a laboratory three degrees of freedom lumped mass system with a piecewise-linear spring. The output degree of freedom is located away from the nonlinearity so that the partial feedback linearisation possesses nonlinear internal dynamics. The dynamic behaviour of the linearised part is specified by eigenvalue assignment and an investigation of the zero dynamics is carried out to confirm stability of the overall system. A tuned numerical model is developed for use in the controller and to produce numerical outputs for comparison with experimental closed-loop results. A new limitation of the feedback linearisation method is discovered in the case of lumped mass systems – that the input and output must share the same degrees of freedom.

1. Introduction

The growing demand for increased performance of mechanical and aerospace systems with reduced weight and fewer emissions leads to research initiatives that aim to exploit the characteristics of nonlinear systems. While the control of linear systems is well understood, most engineering systems behave nonlinearly, at least to some degree, and require the application of a nonlinear controller if the system is to behave according to design requirements. Non-smooth nonlinearities such as bi-linearity and freeplay are commonplace in joints and connections, but difficult to treat because of the abrupt changes in dynamic behaviour that occur as parts come into contact and separate. In this paper non-smooth nonlinearity is treated by the method of feedback linearisation [1–3], a nonlinear control method capable of transforming a nonlinear system into a linear one by appropriate choice of input. In complete input-output feedback linearisation all the states of a nonlinear system are linearised. This differs from the more general problem of partial input-output feedback linearisation, in which only the input-output map is linearised and the number of outputs is fewer than the number of states of the system. The remaining part of the system that has not been linearised generally remains nonlinear and is uncontrollable. Therefore its stability must be determined by checking the so-called zero dynamics; equivalent to a linear time invariant (LTI) system being minimum phase when all its zeros are in the left-hand half-plane. The method has found application in numerous engineering fields including

* Corresponding author.
E-mail addresses: domenico.lisitano@polito.it (D. Lisitano), S.Jiffri@liverpool.ac.uk (S. Jiffri), elvio.bonisoli@polito.it (E. Bonisoli), J.E.Mottershead@liverpool.ac.uk (J.E. Mottershead).

https://doi.org/10.1016/j.jsv.2017.11.047

Nomenclature

$\mathbf{A}$	State-space matrix for the zero dynamics
$\mathbf{b}$	Input vector
$\mathbf{C}$	Viscous damping matrix
$\mathbf{C_{nl}}$	Nonlinear damping matrix
f	objective function for linear model optimisation
$\mathbf{f_{Cnl}}$	Nonlinear damping force
$\mathbf{f_{Knl}}$	Nonlinear stiffness force
f_{n}	Desired natural frequency
$\mathbf{f}(t)$	Excitation force
g_1, g_2	left and right gaps: nonlinear spring
$g_{\mathrm{c,i}}$	Parameter correction factor
$\mathbf{g_q}$	Force distribution vector
$\mathbf{K}$	Stiffness matrix
$\mathbf{K_{nl}}$	Nonlinear stiffness matrix
$k_{g,i}$	Stiffness of the spring between ith degree of freedom and the floor
k_{ij}	Stiffness of the spring between ith degree of freedom and jth degree of freedom
$k_{\mathrm{g,nl}}$	Nonlinear stiffness
$\mathbf{H}$	Receptance matrix
l_2	Vertical position of the nonlinear spring slider
$\mathbf{M}$	Mass matrix
m_i	Mass of the ith degree of freedom
n	Relative degree
$\mathbf{q}$	Displacement vector
$\mathbf{T}$	Transformation matrix
$\mathbf{T_{pl}}$	Transformation matrix of the controllable linerised coordinates
t	Time
$u(t)$	Real input
v	Virtual input
$\mathbf{x}$	State space vector
$\mathbf{z}$	Linearised coordinate
$\mathbf{z}_{\mathrm{eq}}$	Equilibrium point
$\mathbf{z}_{\mathrm{id}}$	Internal dynamics
$\mathbf{z}_{\mathrm{zd}}$	Zero dynamics
$\dot{\bullet}$	First derivative with respect to time
$\ddot{\bullet}$	Second derivative with respect to time
$(\tilde{\bullet})$	Parameter nominal value
α	Viscous damping coefficient for mass proportionality
β	Viscous damping coefficient for stiffness proportionality
ε	Nonlinear damping force coefficient
η	Degree of freedom location of the nonlinearity
$\Omega(t)$	Time variant frequency for sweep excitation
ω_{n}	Desired natural frequency
χ	Output degree of freedom
ζ_{n}	Desired damping ratio

the following: robotics, to control the trajectory and the body posture of a mobile robot [4–7]; electric motors, to stabilise the position and velocity of the rotor and to control the voltage [8–12]; in fuel cells, to control the pressure of hydrogen and oxygen [13]; and in actuation systems with valve nonlinearities [14,15]. In aerospace engineering the technique is used to control drones [16,17] and to suppress wing flutter [18–20]. All these examples relate to smooth nonlinearities in the system or in the input, which means that there are no non-differentiable points in the nonlinear characteristic. The application of the feedback linearisation control to non-smooth nonlinear systems is an area open to research, possibly because the smoothness of the nonlinearity was originally said to be a requirement for the application of feedback linearisation. Tao and Kokotovic [21] proved this constraint to be unnecessary at least in cases where the non-smooth nonlinearity is in the input and has a dead zone, piecewise, backlash or hysteresis characteristic - for these cases they also developed adaptive methods. Jiffri et al. [22] developed the theory of complete and partial feedback linearisation to nonlinear aeroelastic systems with structural non-