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An analytical model of a curved beam with a T shaped cross section

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ABSTRACT

This paper derives a comprehensive analytical dynamic model of a closed circular beam that has a T shaped cross section. The new model includes in-plane and out-of-plane vibrations derived using continuous media expressions which produces results that have a valid frequency range above those available from traditional lumped parameter models. The web is modeled using two-dimensional elasticity equations for in-plane motion and the classical flexural plate equation for out-of-plane motion. The flange is modeled using two sets of Donnell shell equations: one for the left side of the flange and one for the right side of the flange. The governing differential equations are solved with unknown wave propagation coefficients multiplied by spatial domain and time domain functions which are inserted into equilibrium and continuity equations at the intersection of the web and flange and into boundary conditions at the edges of the system resulting in 24 algebraic equations. These equations are solved to yield the wave propagation coefficients and this produces a solution to the displacement field in all three dimensions. An example problem is formulated and compared to results from finite element analysis.

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1. Introduction

Beams are structural elements that are designed to resist an external force and are usually used to reinforce a structure. They are typically designed to resist loads that are normal to their undeformed beam axis. Beams are constructed using a large variety of cross sectional shapes and can have curvature or are straight along their major axis. The first analytical model of a beam was developed by Bernoulli and Euler [1] and this equation is presented in almost every text on mechanical vibrations or mechanical wave motion. This theory uses the assumption that plane cross-sections remain plane and perpendicular to the deformed beam axis when the beam is undergoing bending initiated by a normal force. Timoshenko [2] revised this work so that the rotation angle of the neutral axis of the beam was a function of the polar inertia and shear deformation. Both the Bernoulli-Euler and Timoshenko beam theories exist for straight and curved beams. Higher order displacement functions have been added to beam theory, notably by Bickford [3] who used a third-order polynomial through the thickness of the

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beam to model the in-plane displacement field and Karama et al. [4] who used an exponential function to model the shear distribution in the beam.

Ambati [5] analyzed and discussed the in-plane response of annular rings using elasticity theory. Kirkhope [6] derived the stiffness and inertia matrices for thick circular rings using an energy method. Hawkings [7] developed a theory of inextensional vibrations of a circular ring where the principal axes of inertia of the cross section did not lie in the ring plane. Kirkhope et al. [8] analyzed the vibration of closed uniform rings with unsymmetrical cross sections. Bhimaraddi [9] developed a shear deformable theory for curved beams of constant curvature by assuming a parabolic variation for the shear strains. Lin and Lee [10] studied curved Timoshenko beams with generalized boundary conditions. The preceding references are either elastic response for a uniform beam or lumped inertial and stiffness response for a beam of varying cross section. These lumped responses tend to be lower frequency models, defined here as up to 1 kHz. Numerous researchers have developed numerical methods to model curved beams. Leung and Zhu [11] derived *p*-elements for in-plane vibrations of thin and thick curved beams. Gimena et al. [12] derived a three dimensional curved beam element with a cross section whose area varied. Some investigations of curved beams with varying material properties have been completed. Pydah and Sabale [13] investigated the static response of a functionally graded circular beam using Bernoulli-Euler theory. Wang and Liu [14] formulated an elastic response to a curved beam with orthotropic functionally graded layers. Rothwell [15] derived approximate formulas of flange efficiency for curved beams with varying cross sections.

This paper develops an analytical model of a circular T shaped beam for mid frequency range analysis. The problem is modeled using the two-dimensional plane stress elastic equations for in-plane motions of the web, classical plate equation for the out-of-plane motion of the web and Donnell shell equations for three-dimensional motion of the flange. The method presented here combines these components into a single model that can predict the response of a T beam to various external loads. This new method allows significant compliance across the web coupled to the dynamic effects of the flange, which allows accurate analytical predictions at higher frequencies than previously available. For the specific example presented in this paper, the three dimensional displacement fields are studied with respect to independent loads in the three primary axes of the cylindrical coordinate system. The frequency limitations of the new model are material and geometry dependent, however, this paper includes a comparison of the new model to solutions generated using finite element modeling for a specific geometry and discussions about the frequency range of agreement.

2. System model

The system under consideration is a circular closed beam with a T shaped cross section. The outer narrow component is called the web and the inner wide component is called the flange. The structure is excited with continuous spatial and time harmonic excitation applied to the outer edge of the web. A schematic of this system showing the dimensions and the web coordinate system is shown in Fig. 1. The web coordinate system is a function of the independent variables radial direction, angular direction and time. The model of the web has no spatial extent in the longitudinal direction. The flange coordinate system has the same orientation as the web coordinate system, however, it is a function of the independent variables angular

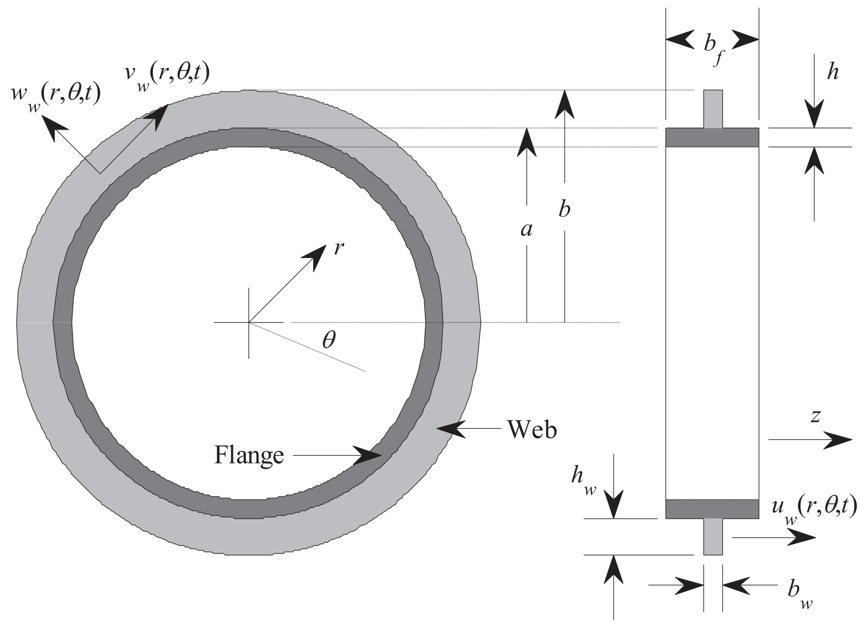

Fig. 1. Circular beam with a T shaped cross section showing dimensions and the web coordinate system.