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Multiple scattering and stop band characteristics of flexural waves on a thin plate with circular holes

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ABSTRACT

A numerical procedure is proposed for the multiple scattering analysis of flexural waves on a thin plate with circular holes based on the Kirchhoff plate theory. The numerical procedure utilizes the wave function expansion of the exciting as well as scattered fields, and the boundary conditions at the periphery of holes are incorporated as the relations between the expansion coefficients of exciting and scattered fields. A set of linear algebraic equations with respect to the wave expansion coefficients of the exciting field alone is established by the numerical collocation method. To demonstrate the applicability of the procedure, the stop band characteristics of flexural waves are analyzed for different arrangements and concentrations of circular holes on a steel plate. The energy transmission spectra of flexural waves are shown to capture the detailed features of the stop band formation of regular and random arrangements of holes. The increase of the concentration of holes is found to shift the dips of the energy transmission spectra toward higher frequencies as well as deepen them. The hexagonal hole arrangement can form a much broader stop band than the square hole arrangement for flexural wave transmission. It is also demonstrated that random arrangements of holes make the transmission spectrum more complicated.

1. Introduction

Propagation characteristics of flexural waves on thin plates have been studied extensively in the foregoing literature for their significant implications in the design of engineering structures and equipments. The interaction of flexural waves with different types of inclusions in plate structures, i.e., holes (cutouts), patches, inhomogeneities with different material properties or thicknesses, has been of particular interest regarding the dynamic stress concentration around them leading to possible structural failures [1] and the damage detection using elastic guided waves [2]. Furthermore, when these inclusions are distributed on a plate, the overall propagation characteristics of flexural waves are altered. Recently, the phononic or metamaterial plate design by introducing artificial arrangements of inclusions is attracting increasing attention [3].

Problems of flexural wave scattering by a single inclusion have been studied by many authors. While the exact analysis of such problems needs to be carried out based on the three-dimensional theory of elastodynamics, approximate plate theories

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have also been employed. Since the work of Pao and Chao [4] who analyzed the flexural wave scattering by a circular cavity, the Mindlin theory has been adopted by many other investigators [5–8]. Furthermore, when a plate is sufficiently thin, the classical Kirchhoff plate theory can be used to a good approximation to analyze the flexural wave scattering by inclusions of various types [9–13]. The validity of the results predicted by the Kirchhoff plate theory has been demonstrated by comparison with the results of the exact theory [11] and with experiments [14].

In contrast to the problems of flexural wave scattering by a single inclusion, the corresponding studies for multiple inclusions are more complicated and relatively scarce in the literature. The foregoing works include those of Lee and Chen using the null-field integral equation method [15] and the multipole method [16], although the numerical examples demonstrated by them are limited to the scattering by two inclusions. Peng [17] applied the so-called acoustic wave propagator method to analyze the multiple scattering of a flexural wave by nine cylindrical patches on a plate. Some investigators have analyzed the averaged propagation characteristics of flexural waves due to multiple scattering by random distributions of inclusions. Namely, Weaver [18] analyzed the multiple scattering of the flexural wave by random sprung masses. Dixon and Squire [19] analyzed the energy transport velocity of the flexural wave on a random plate with circular inclusions. Parnell and Martin [20] obtained the effective wavenumber of the flexural wave for a plate with random inclusions based on the quasi-crystalline approximation. On the other hand, when the inclusions are arranged periodically, the plate can exhibit so-called bandgaps or stop bands for the flexural wave propagation. Movchan et al. [21] analyzed the Bloch-Floquet waves in a plate with a periodic array of circular holes, and demonstrated the dispersion relation of the flexural wave including the bandgaps. Recently, Cai and Hambric [22] analyzed the multiple scattering of the flexural wave by 21 circular inclusions arranged on a square lattice and the corresponding stop band formation. The works mentioned above [15–22] were all based on the Kirchhoff plate theory.

As mentioned above, the understanding of the multiple scattering and the stop band formation of flexural waves is important for the design of phononic or metamaterial plates as it opens a way to the suppression, guiding, focusing, etc, of flexural waves. In the ideal case of an infinitely extended periodic array of inclusions, the flexural wave propagation is either allowed (pass band) or forbidden (stop band) depending on the frequency, according to the Bloch-Floquet theory [21]. In realistic situations, it is often necessary to analyze the stop band formation by a finite number of inclusions arranged on a plate. For periodic microstructures which are extended over a finite length in the propagation direction, the wave transmission spectrum shows complicated oscillatory features in addition to forming stop bands, as demonstrated for the wave transmission in multilayered structures [23] and fiber-reinforced media [24]. Therefore, a complete understanding of stop band phenomena of flexural waves necessitates a numerical method for the multiple scattering analysis which can account for a large number of inclusions arranged on the plate in an arbitrary manner.

In this paper, a general numerical procedure is presented to analyze the multiple scattering of flexural waves on a thin plate with arbitrary arrangements of circular holes based on the Kirchhoff plate theory. The time-harmonic deflection field of the plate is given as the sum of the incident wave and the scattered waves by the holes, which are expressed by the wave function expansions used in other foregoing works [22]. Instead of using the commonly employed expressions of the so-called Graf addition theorem, the present procedure employs the numerical collocation technique to construct a linear system of algebraic equations to determine the expansion coefficients directly, by following the computational multiple scattering studies for fiber-reinforced composites [25,26]. Furthermore, the periodicity of the hole arrangement is assumed perpendicular to the propagation direction of the incident wave in order to save computational costs. Using the proposed method, the multiple scattering and the stop band formation of flexural wave is analyzed for different arrangements of circular holes with different concentrations.

2. Formulation

2.1. Governing equations of flexural motions of a thin plate

Consider a thin, infinitely extended, isotropic and linear elastic plate containing N non-overlapping, through-thickness, circular holes (radius R) which are arranged arbitrarily in the x-y plane as shown in Fig. 1. The position vector of an arbitrary point on the plate is denoted by $\mathbf{r}$, and the position vector of the center of the lth hole ($l = 1, 2, \ldots, N$) is denoted by $\mathbf{r}_l$. According to the Kirchhoff plate theory, the transverse motion of a thin plate without external loads is governed by [27]

$$D\nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} = 0, \tag{1}$$

where w is the transverse displacement (deflection) of the plate, t is the time, h is the plate thickness, ρ is the mass density, $D = Eh^3/\{12(1-\nu^2)\}$ is the flexural rigidity of the plate, E is Young's modulus, and ν is Poisson's ratio. The two-dimensional bi-harmonic operator is defined by $\nabla^4 = \nabla^2\nabla^2$, where ∇^2 is the Laplacian operator. A time-harmonic solution of Eq. (1) of a form $w = We^{-\mathrm{i}\omega t}$ is considered in this paper, where W is a complex-valued function of the position, i is the imaginary unit and ω is the angular frequency. The function W satisfies the bi-Helmholtz equation