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Veering and nonlinear interactions of a clamped beam in bending and torsion

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ABSTRACT

Understanding the linear and nonlinear dynamic behaviour of beams is critical for the design of many engineering structures such as spacecraft antennae, aircraft wings, and turbine blades. When the eigenvalues of such structures are closely-spaced, nonlinearity may lead to interactions between the underlying linear normal modes (LNMs). This work considers a clamped-clamped beam which exhibits nonlinear behaviour due to axial tension from large amplitudes of deformation. An additional cross-beam, mounted transversely and with a movable mass at each tip, allows tuning of the primary torsion LNM such that it is close to the primary bending LNM. Perturbing the location of one mass relative to that of the other leads to veering between the eigenvalues of the bending and torsion LNMs. For a number of selected geometries in the region of veering, a nonlinear reduced order model (NLROM) is created and the nonlinear normal modes (NNMs) are used to describe the underlying nonlinear behaviour of the structure. The relationship between the 'closeness' of the eigenvalues and the nonlinear dynamic behaviour is demonstrated in the NNM backbone curves, and veering-like behaviour is observed. Finally, the forced and damped dynamics of the structure are predicted using several analytical and numerical tools and are compared to experimental measurements. As well as showing a good agreement between the predicted and measured responses, phenomena such as a 1:1 internal resonance and quasi-periodic behaviour are identified.

1. Introduction

An important consideration for structures operating in dynamic environments is the occurrence of closely spaced eigenvalues of the linear normal modes (LNMs) of vibration. This occurrence is particularly significant when eigenvalues become a function of operating conditions. For instance, closely-spaced eigenvalues can have a significant effect on the aeroelastic behaviour of wings since the 'closeness' of the eigenvalues affects the velocity at which the onset of flutter occurs [1]. Similarly, closely-spaced eigenvalues can strongly influence the dynamic response of a nonlinear system, and may result in strong internal resonances between the underlying LNMs and other nonlinear dynamic behaviour [2]. In this context, understanding the nonlinear interaction of LNMs, and how this interaction is influenced by the of the change of eigenvalues, is critical for many engineering structures.

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In near-symmetric linear systems, the eigenvectors associated with closely-spaced eigenvalues can be highly sensitive to perturbations in the symmetry of the physical structure as described in Ref. [3]. Over larger perturbations in the symmetry, the phenomenon of mode veering can be observed [4,5]; resulting in the separation of the eigenvalues and the correlation of the eigenvectors (e.g. off-axis terms in the modal assurance criterion [6] will appear). Examples of mode veering in the static perturbation of structures have been shown in a pressure vessel [7], cable dynamics [8,9], a pre-stressed frame [10], and imperfect beams [11] with additional work connecting mode veering and mode localisation [12]. In contrast to the mode veering phenomenon, if a symmetry preserving change of geometry is applied to a structure, the closely-spaced eigenvalues will cross instead of veer [5], and no correlation will be observed between the eigenvectors.

It is well known that, for nonlinear systems, internal forces can cause an exchange of energy between nearly commensurate LNMs of vibration; termed internal, or auto-parametric, resonances. An in-depth line of work examining 1:2 resonances in a tuneable cantilever beam-mass system is detailed in Ref. [13], for example. For structures exhibiting closely-spaced eigenvalues, 1:1 internal resonances have been observed in the dynamics of symmetric systems with cubic nonlinearities [14] as well as stretched strings, beams, plates, and rotating disks as discussed by Nayfeh in Ref. [15], for example. Additionally, it has been shown that as physical parameters of stretched cables and symmetric shallow arches are changed, whilst preserving symmetry, a crossing occurs between uncorrelated natural frequencies (i.e. symmetric and anti-symmetric LNMs). At the point of crossing, a 1:1 internal resonance can be realized if the system of interest contains the proper orthogonality conditions discussed in Ref. [16]. Again, if the symmetry is broken through a change in physical parameters, the natural frequencies will veer instead of cross. Lacarbonara et al. [11] investigated the nonlinear dynamics of an imperfect beam at veering, finding 1:1 internal resonances; however, only coupled motions of the modes of vibration were physically realized in the vicinity of veering contrary to the perfect beams investigated in Ref. [16], where the interaction between the linear modes was not activated.

The veering/crossing phenomena emphasises the importance of the inertial and stiffness distribution in structures with closely spaced eigenvalues. In dynamic linear systems, the inertial and stiffness properties of a structure is described using LNMs (i.e. eigenvalues and eigenvectors). A perturbation of either property directly affects the eigenvalues and eigenvectors, and veering/crossing can be observed in the correlation between the eigenvectors. In dynamic nonlinear systems, nonlinear normal modes (NNMs) of vibration [17] are used to describe the inertial and stiffness properties of a structure. As a nonlinear system experiences a large amplitude of deformation, there is a potential for the effective mass and/or stiffness to change based on the mechanism of the nonlinearity. In continuous geometrically nonlinear systems, the change in effective mass and/or stiffness can be observed in the NNM backbone curves, i.e. the loci of NNM responses, as discussed in Ref. [18]. A deeper understanding of the dynamic motion is also be obtained by projecting the NNMs onto the underlying LNMs of vibration providing an indication of the activation of the nonlinear interaction [19,20]. Nonlinearity often causes shifts in the response frequencies, which can have the effect of tuning the dynamic behaviour, leading to a nonlinear analogue of the linear veering behaviour.

This paper considers a clamped-clamped beam with movable masses, which allows the symmetry of the structure to be broken, and for the torsional inertia to be tuned. This not only enables the linear crossing and veering phenomena to be investigated, but also its influence on the nonlinear behaviour of the beam, which arises from dynamic tension effects. Nonlinear veering is demonstrated between closely-spaced eigenvalues of the fundamental bending and torsion LNMs of the beam. Veering between the fundamental LNMs is demonstrated using a full-order model in Abaqus® in section 2. In section 3, nonlinear interactions between two LNMs are identified using NNMs calculated from nonlinear reduced order models (NLROMs). The NLROMs used in this investigation are determined using static load cases and the geometric nonlinear capabilities of Abaqus® as described in Refs. [21,22]. The resulting NNMs illustrate the undamped amplitude-dependent nonlinear behaviour between the two LNMs of interest and draw a connection back to the linear veering phenomena. Specifically, it is shown that, as amplitude increases, the initially bending-dominated NNM shifts in frequency more readily than that dominated by torsion. This results in the response frequency of the bending-dominated frequency approaches that of the torsion-dominated LNM. In turn, this leads to NNM responses composed of both bending and torsional components. This nonlinear veering-like behaviour requires both an asymmetry in the structure, and closely-spaced natural frequencies. In section 4, analytical and numerical tools are used to find the forced-damped dynamics of the NLROM. This reveals nonlinear phenomena such as 1:1 internal resonances as well as fold and torus bifurcations. Finally, in section 5, a comparison is made between the forced responses of the resulting NLROM and the experimentally measured response from a slow sine chirp to validate the behaviour found in the numerical study.

2. Structure and model description

2.1. Physical description

The structure under consideration exhibits close natural frequencies between the first bending and first torsion LNMs, and consists of two beams joined in the middle as shown in Fig. 1a. The main beam is clamped at both ends and is joined in the middle to a cross-beam, which has concentrated masses attached at both ends. The concentrated masses are adjustable permitting the change of torsional inertia with limited influence on the bending inertia of the system (i.e. a change in L_1 and L_2 shown in Fig. 1a). A finite element model was created in Abaqus® to establish a high degree of freedom (DOF) linear model. A total of 288 B31 beam elements, (6 DOFs at each node [22]) were used to discretise the cross section of each beam resulting in