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Bryan's effect and anisotropic nonlinear damping

Stephan V. Joubert*, Michael Y. Shatalov, Temple H. Fay, Alexander V. Manzhirov [1]

Department of Mathematics and Statistics, Tshwane University of Technology, South Africa

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A B S T R A C T

In 1890, G. H. Bryan discovered the following: "*The vibration pattern of a revolving cylinder or bell revolves at a rate proportional to the inertial rotation rate of the cylinder or bell.*" We call this phenomenon *Bryan's law* or *Bryan's effect*. It is well known that any imperfections in a vibratory gyroscope (VG) affect Bryan's law and this affects the accuracy of the VG. Consequently, in this paper, we assume that all such imperfections are either minimised or eliminated by some known control method and that *only damping is present* within the VG. If the damping is *isotropic* (linear or nonlinear), then it has been recently demonstrated in this journal, using symbolic analysis, that *Bryan's law remains invariant*. However, it is known that linear *anisotropic* damping *does* affect Bryan's law. In this paper, we *generalise Rayleigh's dissipation function* so that *anisotropic* nonlinear damping may be introduced into the equations of motion. Using a mixture of numeric and symbolic analysis on the ODEs of motion of the VG, for *anisotropic light nonlinear damping*, we demonstrate (up to an approximate average), that *Bryan's law is affected* by any form of such damping, causing pattern drift, compromising the accuracy of the VG.

1. Introduction

Bryan's law states the following:

> *The vibration pattern of a revolving cylinder or bell revolves at a rate proportional to the inertial rotation rate of the cylinder or bell.*

This law was discovered by G. H. Bryan in 1890 [1]. The proportionality constant mentioned in the law is called Bryan's factor - it is used to calibrate a vibratory gyroscope (VG).

The effects of manufacturing imperfections and flaws in VG have been discussed in many publications. It is known that damage (cracks) and/or nonaxisymmetry and/or mass and/or stiffness imperfections, cause a frequency split in a VG, affecting Bryan's effect and decreasing accuracy (see, e.g. Refs. [2–8]). Furthermore, it is *known* that *anisotropic linear damping* within the VG affects Bryan's law ([9,10]), but it is *not known* whether anisotropic quadratic damping (and higher orders of such damping) affects Bryan's law.

* Corresponding author.
E-mail addresses: joubertsv@tut.ac.za (S.V. Joubert), manzh@inbox.ru (A.V. Manzhirov).

[1] Permanent address: Institute of Problems in Mechanics, Russian Academy of Sciences, Russia.

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Assuming that there are no imperfections in the body of the VG that might cause a frequency split, we show how to introduce *any form of anisotropic nonlinear light damping* into the equations of motion of a VG. These results, with no detailed derivation (unlike our work below), were used in a chapter of the book [11] in order to demonstrate the effectiveness of the electronic control system introduced there. In order to achieve the results presented in this paper, our assumptions allow us to obtain an approximation for a *generalised Rayleigh's dissipation function*. This allows us to *symbolically analyse* approximated ODEs of motion via the method of averaging. This symbolic analysis, supported by numerical (graphical) analysis, demonstrates that any *form of anisotropic nonlinear light damping does affect Bryan's law*, causing pattern drift, affecting the accuracy of the gyroscope. This is in sharp contrast to the fact that *no form of isotropic light damping affects Bryan's law* (as demonstrated symbolically in this journal [12]).

Notation is discussed in Section 2, while Sections 3 and 4 introduce a generalised version of Rayleigh's dissipation function that may be modified to model nonlinear tangentially anisotropic damping. In Section 5, fast variable ODEs of motion are obtained that model combinations of nonlinear anisotropic damping of any order. In Section 6, the system of fast variable ODEs is transformed into slow variable ODEs. Averaged ODEs of motion are obtained in Section 7 where graphical comparisons are made. In Section 8, the averaged ODEs are analysed quantitatively, yielding further insight into nonlinear anisotropic lightly damped VG behaviour. Conclusions are drawn in Section 9.

2. Notation

For completeness, we repeat some of the notation of [12], because this article is a continuation of the work started there. Indeed, in this paper we consider an annular *disc vibratory gyroscope (DVG)* with outer radius q and inner radius p, using the polar coordinate system $Or\varphi$ (see Fig. 1 of [12]). We do this because all of the characteristics of a VG may be illustrated in a technically easy manner when compared to the calculation details of a more complicated VG structure such as [7] for a bell-shaped system and [13] for a paraboloid system. In general, we might have considered a curvilinear coordinate system $Or\varphi k$ where k is the axis of symmetry variable, r the radial variable and φ the tangential variable.

We consider the inertial angular rate of rotation $\varepsilon\Omega$ about the axis of symmetry to be small in the sense that it is *substantially smaller* than the lowest eigenvalue ω_0 of the vibrating system, allowing us to neglect $O(\varepsilon^2)$ terms.

Assume that u is radial, v is tangential and w is axial displacement for the particle P in the VG being considered (see, e.g., Fig. 1 of [12]), with

$$u = U[C(t)\cos m\varphi + S(t)\sin m\varphi], \tag{1}$$

$$v = V[C(t)\sin m\varphi - S(t)\cos m\varphi], \tag{2}$$

$$w = W[C(t)\cos m\varphi + S(t)\sin m\varphi], \tag{3}$$

where U, V and W are eigenfunctions of one or two variables appropriate to the coordinate system having the dimensions of length and m is the vibration mode number or circumferential wave number. The functions $C(t)$ and $S(t)$ are dimensionless functions of time t. For a ring or annulus, we would take $w = 0$ and assume that all unbalanced forces in the axial direction are zero. In ([14]) it was demonstrated that for a nonrotating DVG, the eigenfunctions $U = U(r)$ and $V = V(r)$ remain invariant under slow rotation. Consequently, U and V may be calculated accurately using a numerical routine as demonstrated in Ref. [15] or, more tediously, in terms of Bessel and Neumann functions.

For our DVG, the equations of motion are similar to those for more complicated structures such as those derived in Ref. [16] Eq. (19), where Bryan's law for a layered planet and [7] Eq. (20) where a bell-shaped VG were examined.

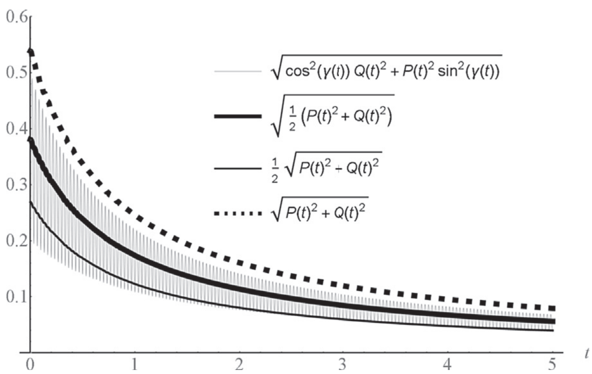

Fig. 1. The RMS $\sqrt{\frac{P^2+Q^2}{2}}$ appears to be a good average for the expression $\sqrt{(P^2 \sin^2\gamma + Q^2\cos^2\gamma)}$.