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On modal cross-coupling in the asymptotic modal limit

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ABSTRACT

The conditions under which significant modal cross-coupling occurs in dynamical systems responding to high-frequency, broadband forcing that excites many modes is studied. The modal overlap factor plays a key role in the analysis of these systems as the modal density (the ratio of number of modes to the frequency bandwidth) becomes large. The modal overlap factor is effectively the ratio of the width of a resonant peak (the damping ratio times the resonant frequency) to the average frequency interval between resonant peaks (or rather, the inverse of the modal density). It is shown that this parameter largely determines whether substantial modal cross-coupling occurs in a given system's response. Here, two prototypical systems are considered. The first is a simple rectangular plate whose significant modal cross-coupling is the exception rather than the norm. The second is a pair of rectangular plates attached at a point where significant modal cross-coupling is more likely to occur. We show that, for certain cases of modal density and damping, non-negligible cross coupling occurs in both systems. Under similar circumstances, the constraint force between the two plates in the latter system becomes broadband. The implications of this for using Asymptotic Modal Analysis (AMA) in multi-component systems are discussed.

1. Introduction and motivation

The study of structures responding to high-frequency, broadband forces that excite many modes (a scenario called "the asymptotic modal limit") has been ongoing since the advent of Statistical Energy Analysis (SEA) as discussed by Lyon, Maidanik, et al. [1,2]. Also see the second edition of Lyon's comprehensive text on the subject: "Statistical Energy Analysis of Dynamical Systems: Theory and Application" [3]. Note that SEA and similar techniques use a "white noise" approximation of broadband excitation, which, as Elishakoff notes, is insufficient in fracture mechanics [4]. In this work with orthotropic plates [4], Elishakoff outlines a technique for estimating plate response in such a limit, noting that real-world plate behavior is a consequence of an "intermediate between all-round simple support and all-round clamping". In his review of SEA, Fahy identified that one of the motivating factors for statistical energy analysis was the design of rockets, where he reported that approximately 500,000 modes exist below 2 kHz in the Saturn V [5]. Even today, modeling that many high-frequency modes is prohibitive in terms of computational cost. Classical Modal Analysis (CMA) would require very short time steps, and the Finite Element Method (FEM) would additionally require incredibly fine meshes. Lyon suggested that if equipartition of energy among responding modes could be assumed, the response of simple systems could be easily estimated [2].

Soon after, Dowell and Kubota proposed Asymptotic Modal Analysis (AMA), a reduction of CMA as the number of excited modes in a system becomes large [6]. AMA leverages work in statistical mechanics by Crandall [7,8] to separate the spatial contribution of mode shapes to a response from the frequency information contribution. Moreover, it uses the work

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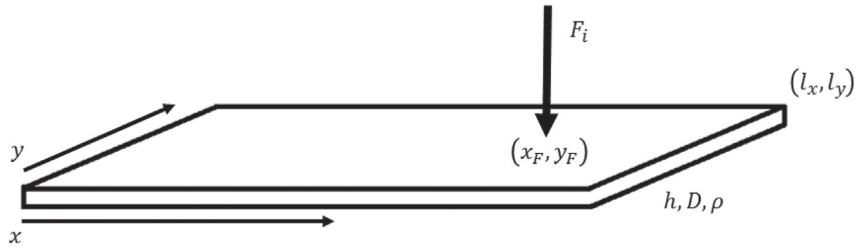

Fig. 1. Schematic of a single rectangular plate experiencing a high-frequency, broadband point excitation.

of Bolotin [9,10] who studied the asymptotic behavior of elastic systems in the frequency domain. In his work, Bolotin found that for elastic systems such as plates, when the dominant response frequencies are large relative to the fundamental natural frequency of the system, the boundary conditions contribute very little to the ultimate modal behavior. This justified the application of AMA to systems with many different boundary conditions. In 1998, Dowell and Tang extended their work to a plate carrying a concentrated mass/spring system [11]. Most recently, AMA has been used to analyze nonlinear and coupled dynamical systems [12,13].

In order for systems responding in the asymptotic modal limit to be analyzed using AMA, resonant peaks must be well-separated. When these peaks overlap, an interaction called modal cross-coupling occurs which offers significant challenges to both AMA and SEA. In a computational study, Li and Dowell investigated modal cross-coupling for a rectangular aluminum plate [14]. They found that, for most configurations of the plate, the resonant peaks remain well-separated. However, this may not be the case for all structures, particularly those with a high modal density and/or large damping. The relationship between these two system characteristics and some of the consequences of that relationship are studied in the literature, including in the work of Fahy and Mohammed [15]. In their work, the sensitivity of the Coupling Loss Factor (CLF) to the modal overlap factor μ in the context of Statistical Energy Analysis (SEA) of coupled systems (such as two connected beams or two connected plates) is studied. The modal overlap factor is commonly used in recent investigations in vibroacoustics, such as the work of Denis et al. [16] where the presence of an Acoustic Black Hole on a beam dramatically increases its modal overlap factor. Other effects of varying the modal overlap factor have been studied, such as changes in the standard deviation of energy density in the work of Langley [17]. And finally, the modal overlap factor is used to determine the applicability of reduced order models in situations where many modes are excited in a system. It is commonly noted in the literature that these models, such as SEA, are applicable when $\mu \gg 1$ [18].

In this work, configurations of dynamical systems that experience significant modal cross-coupling are explored. More specifically, the sensitivity of modal cross-coupling to the modal overlap factor is quantified. In section 2, the problem of a single rectangular plate responding in the asymptotic modal limit is revisited. The system behavior for well-separated and cross-coupled is illustrated. Moreover, some physical criteria necessary for triggering significant cross-coupling are explored. In section 3, the effect of cross-coupling in multi-component systems is studied. It is important to note that, even if individual components have well-separated resonant peaks in a given excitation band, the coupled system may not. For example, if component 1 has ΔM_1 modes, component 2 has ΔM_2 modes, etc., then the coupled system has $\sum_i \Delta M_i$ modes in the same excitation band. Those modes may interact, and recognizing the influence of cross-coupling in those situations can be very important, especially in a scenario where a system has 250 modes for every Hz as in the Saturn V rocket.

2. A single rectangular plate

We consider a rectangular plate experiencing high-frequency, broadband forcing that excites many modes, illustrated in Fig. 1. Note that a two-dimensional prototypical example is used because the modal density of a one-dimensional system (a beam) drops off rapidly as frequency increases. As a consequence, it is difficult to analyze beams with AMA.

According to Li and Dowell [14], the mean-square response of the plate may be computed by

$$\overline{w}^2 = \int_0^\infty |H_w|^2 \Phi_F \mathrm{d}\omega \tag{1}$$

$$\overline{w}^2 = \sum_m \sum_n \frac{\psi_m(x_F, y_F)\,\psi_n(x_F, y_F)\,\psi_m \psi_n}{M_m M_n} \int_0^\infty \frac{1}{S_m \overline{S}_n} \mathrm{d}\omega \Phi_F \tag{2}$$

where $\overline{w}^2$ is the mean-square of the transverse displacement, H_w is the transfer function for the transverse response, Φ_F is the excitation power spectrum (assumed to be uniform in frequency), and ω is frequency. ψ_m is the mth system mode shape, (x_F, y_F) is the coordinate set for the point excitation, M_m is the modal mass of mode m[1], and S_m is the characteristic equation of mode m

[1] This assumes Kirchoff-Love (KL) thin plate theory. As Goldenveizer [19] and Kaplunov [20] have shown, KL plate theory may break down at higher frequencies. See Appendix B for the derivation of the inertial correction factor described by Goldenveizer for this problem.