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Coupled dynamics of a viscoelastically supported infinite string and a number of discrete mechanical systems moving with uniform speed

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ABSTRACT

The mutual interaction between a number of multi degrees of freedom mechanical systems moving with uniform speed along an infinite taut string supported by a viscoelastic layer has been studied using the substructure synthesis method when base excitations of a common frequency are given to the mechanical systems. The mobility or impedance matrices of the string have been calculated analytically by Fourier transform method as well as wave propagation technique. The above matrices are used to calculate the response of the discrete mechanical systems. Special attention is paid to the contact forces between the discrete and the continuous systems which are estimated by numerical simulation. The effects of phase difference, the distance between the systems and different base excitation amplitudes on the collective behaviour of the mechanical systems are also studied. The present study has relevance to the coupled dynamic problem of more than one railway pantographs and an overhead catenary system where the pantographs are modelled as discrete systems and the catenary is modelled as a taut string supported by continuous viscoelastic layer.

1. Introduction

Study of dynamics of a continuum resting on elastic and viscoelastic foundations with moving loads [1] has been a research topic of considerable interest for several years because of its relevance to a number of problems of practical importance. Railway tracks, bridges, overhead catenary, ropeways are examples of continua that are periodically supported and subjected to point or distributed moving loads [2–5]. Literature available in this field shows that simple models of the continuum are able to capture the basic dynamic features of the real complex systems quite well. In most of the cases, the objective of the study has been to determine the response of the continuum under the application of given loads. Dieterman and Metrikine [6] carried out research on steady state displacement of a beam resting on a viscoelastic foundation. Gavrilov [7] analytically as well as numerically investigated a similar problem with string under the action of accelerating load and studied the string behaviour when the load speed crosses the sonic speed. Metrikine [8] studied the steady state response of such kind of string supported by non-linear viscoelastic layer by means of phase plane analysis. The previous research was further extended by Metrikine and Bosch [9] to study the response of an effective one level catenary as well as a two-level one.

The use of Green's function to solve this kind of problems is also a common approach [10]. Mazilu [4] used Green's function method to show that a stationary load applied to a string produces a response similar to that of a single degree of freedom (dof) system and an increase in the load speed decreases the resonance frequency of the system. A similar method was also used by

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a few authors for finding the response of beams with different boundary conditions [3,11,12].

The vibration induced by a single moving mass along an infinite continua resting on elastic foundation has also been studied [13,14]. While Duffy [13] discussed about the resonance of the system considering beam on a Winkler support as the continuum, asymptotic solution for that kind of problem considering the 'wave drag on the motion' was found out by Gavrilov [14] in case of a string. It can also generate 'trapped mode of oscillation' in the continuum while moving at a constant speed or with a small acceleration [15] and also in case of changing mass [16].

The dynamics of continua when subjected to a moving discrete mechanical system has also been studied by quite a few researchers. Kruse et al. [17] studied the eigenfrequencies of a 2-dof discrete system uniformly moving on a viscoelastically supported string. They found out that some eigenfrequencies can be uniquely determined while some are not depending upon the system parameters. It has also been reported in some research works that the system may experience instability if the discrete mechanical system moves faster than the phase velocity of transverse waves in the continuum [18–22]. In a paper by Metrikine [23] the physical phenomenon of instability has been thoroughly discussed. The system becomes unstable due to the presence of anomalous Doppler waves which generate 'negative radiation damping' at the point of interaction between the continuum and the mechanical systems. These waves start propagating only when the speed of the moving system exceeds the phase velocity. In another paper a detailed discussion on the effect of system parameters on its instability has been presented by Metrikine and Verichev [24].

The main objective of the present research is to study the mutual interaction between a number of discrete multi-dof mechanical systems all travelling with the same axial speed and harmonically excited at the base while keeping contact with a continuum modelled as a viscoelastically supported string. This study stems out of authors' interest in the dynamics of the pantographs used in locomotives for power collection. The harmonic base excitation can also be considered as a very simple assumption of track induced base oscillation. Pantograph-catenary coupled dynamics has been extensively studied over the years. Various simplified models have been proposed for studying this enormously complex problem [25–32]. In an interesting paper, Metrikine and Bosch [9] suggested a model of a single level catenary system as a string on viscoelastic layer and pantograph as single point load. A railway catenary is a bi-periodic structure consisting of two heavily tensed wires, namely, the messenger or carrier wire and the contact wire. If the transverse oscillation of the upper wire is neglected because of wide mismatch between the wave propagation speeds in the two wires, and if the excitation frequency is considered low enough so that it excites large wavelength oscillation in the contact wire, the catenary can be modelled as a single wire supported over a viscoelastic layer. The effects of individual droppers get averaged out in the large wavelength allowing the discrete droppers to be replaced by a viscoelastic layer. In this paper, we study the interaction between the one dimensional continuum and the mechanical system having three degrees of freedom. The problem becomes complicated when more than one discrete system interacts with the string since one system can affect the other in many ways. The speed of the system is kept much less than the phase velocity of transverse waves in the string to avoid the formation of anomalous Doppler waves and subsequently instability to the mechanical systems. Also at high speed the validity of the model to simulate catenary-pantograph interaction becomes questionable. The coupling between the several systems through the string has been considered in the present paper. Although interaction between a moving discrete mechanical oscillator and a taut string has been solved by various researchers using different methods, the problem of mutual interaction between several oscillators mediated through the continuum has not been given enough attention in spite of its importance in many practical fields. To the authors' knowledge, mutual interaction between two point loads has been studied for non-linear viscoelastic foundation [8], however, this analysis is yet to be extended for discrete dynamical systems. Thus an arbitrary number of oscillators has been chosen to study the aforesaid problem, but with linear viscoelastic layer.

In this present work, the coupled system is analysed by the method of substructuring. The response of the string is determined first by replacing the discrete systems with the same number of moving loads. The expressions for mobility and driving point impedance are then calculated. This helps in finding out the responses of the dynamic systems considering a continuous contact with the string. The dynamical systems are considered to be driven harmonically with displacement excitation. Special attention has been given to the calculation of contact forces between the upper masses of the systems and the string. The effect of phase lag between the base excitation provided to the system as well as the separation distance between the oscillators has been taken into account. Finally, the effect of damping property of the viscoelastic layer has been analysed. The proposed model and the analysis method are expected to be useful in studying the catenary-pantograph interaction dynamics in railways.

2. Mathematical model

Consider a heavy taut string with tension T supported by a viscoelastic layer as shown in Fig. 1. The support is modelled as a continuous medium which has stiffness and damping per unit length as $\overline{k}$ and $\overline{c}$, respectively. The mass density of the string that may include the mass of the viscoelastic layer is denoted by μ. A number of 3-dof mechanical systems are considered to move along the string axis with the same uniform speed v, as shown in the figure. It is assumed that the speed of the mechanical systems never exceed the phase velocity of transverse waves in the continuum. This assumption eliminates the possibility of instability in the mechanical systems, which may appear, as reported by several researchers, when the systems move at a speed beyond the phase velocity. It is also assumed that the contact between the string and the topmost mass of the vibrating body is secured for all time. The discrete mechanical systems are excited at the base by harmonic excitation with a common frequency.

The response of the overall system can be calculated by substructure synthesis method. The string together with the viscoelastic support and the discrete mechanical systems are considered to be different substructures.

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