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A robust equal-peak method for uncertain mechanical systems

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ABSTRACT

The linear vibration absorber is a widely-used vibration mitigation device. However, when the absorber is tuned according to Den Hartog's equal-peak method, the resulting narrow bandwidth may decrease its effectiveness, especially when the host structure is uncertain or in the presence of environmental variability. In this paper, a new tuning strategy of the linear absorber, based on the concept of robust equal peaks, is introduced for mitigating a specific resonance of an uncertain mechanical system. Both analytical and numerical investigations are carried out to demonstrate the robustness of the proposed absorber. For 20% uncertainty in the stiffness of the host system, the performance improvement brought by the robust equal-peak method amounts to more than 30% with respect to Den Hartog's tuning rule.

1. Introduction

The use of linear resonators for the mitigation of resonant vibrations was first proposed by Watts [1] and Frahm [2,3] to reduce the rolling motion of ships. The problem was later formalized in more rigorous terms by Ormondroyd and Den Hartog [4], Den Hartog [5] and Brock [6], who developed tuning rules that formed the basis of *Den Hartog's equal-peak method*. The vibration absorber considered in Refs. [4–6] consists of a mass-spring-dashpot system attached to the primary system to be controlled. Through the proper tuning of the absorber's spring and dashpot, it is possible to approximately obtain h_∞ optimization of the frequency response in the vicinity of the target resonance frequency.

Thanks to its simplicity, effectiveness, low cost and small requirements for maintenance [7], the linear vibration absorber (often referred to as tuned mass damper, tuned vibration absorber or dynamic vibration absorber) was implemented in various real-life structures. Its main applications include structures subject to human-induced vibrations, such as spectator stands and pedestrian bridges (the most famous example is the Millenium bridge in London [8]), steel structures excited by machines such as centrifuges and fans, aircraft engines [9], helicopter rotors [10], tall and slender structures subject to wind-induced vibrations, but also power lines [11] and long-span suspended bridges [12,13]. For a list of installations of vibration absorbers in civil structures, the interested reader can refer to [14,15].

An overview of existing designs for passive vibration absorbers is given in Ref. [7]. They include classical absorbers with translational mass movements, pendulum absorbers, centrifugal pendulum absorbers [16], ball absorbers, sloshing liquid absorbers [17] and particle vibration absorbers [18], although the sloshing liquid and particle vibration absorbers have qualitatively different features than the more typical vibration absorbers with a concentrated mass. Many different configurations and variations of the original vibration absorber were studied in the last decades, e.g., a damped host system [19], different combinations of

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response and excitation [20] and the use of multi-vibration absorbers to control several resonances [21].

A fundamental drawback of the linear tuned vibration absorber (LTVA) is that it requires a fine tuning between its natural frequency and the targeted resonance frequency. The LTVA may therefore lose efficiency in the presence of uncertainty [22–24] or nonlinearity [25]. In this context, the main contribution of this paper is to revisit the tuning strategy of a LTVA attached to a primary system with uncertain stiffness and damping. As in previous studies [22,24,26,27], a min-max formulation is adopted; it is solved using the scenario approach [28], a recently-introduced robust optimization method. The careful analysis of the results obtained through the scenario approach brings new insight into the problem which gives rise to the concept of *robust equal peaks* introduced in this paper.

The paper is organized as follows. Section 2 briefly reviews Den Hartog's equal-peak method and the detuning of the LTVA when attached to an uncertain system. In Section 3, the absorber tuning strategy is formulated as a worst-case design problem; the scenario approach is also presented. Section 4 introduces the concept of robust equal peaks and provides numerical and analytical solutions for tuning the robust LTVA. Section 5 discusses the validity and limitations of the proposed tuning strategy. The conclusions of the present study are summarized in Section 6.

2. The linear tuned vibration absorber: equal-peak method

The steady-state response of an undamped mass-spring system subjected to harmonic excitation at a constant frequency can be suppressed using an undamped LTVA, as proposed by Frahm in 1909 [2]. However, LTVA performance deteriorates significantly when the excitation frequency varies. To improve robustness, damping was introduced in the absorber by Ormondroyd and Den Hartog [4]. Denoting by k_1 and c_1 the stiffness and damping of the primary system and by k_2 and c_2 their analogous of the absorber, the equations of motion of the coupled system are

$$\begin{aligned} m_1\ddot{x}_1 + k_1 x_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_2(x_1 - x_2) &= f(t) \\ m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) &= 0 \end{aligned} \tag{1}$$

and their counterpart in the frequency domain is

$$\begin{bmatrix} k_1 + k_2 + i\omega\left(c_1 + c_2\right) - \omega^2 m_1 & -\left(k_2 + i\omega c_2\right) \\ -\left(k_2 + i\omega c_2\right) & k_2 + i\omega c_2 - \omega^2 m_2 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} F(\omega) \\ 0 \end{Bmatrix} \tag{2}$$

where $X_1(\omega)$ and $X_2(\omega)$ are the displacements of the harmonically-forced primary system and of the damped LTVA, respectively. Den Hartog realized that the receptance function of the primary mass, i.e., $h_1 = \frac{X_1(\omega)}{F(\omega)}$, passes through two invariant points independent of absorber damping. He proposed to adjust the absorber stiffness to have two fixed points of equal heights in the receptance curve and to select the absorber damping so that the curve presents a horizontal tangent through one of the fixed points. This laid down the foundations of the so-called *equal-peak method*, widely used in practical applications [29]. As illustrated in Fig. 1, the equal-peak method minimizes the maximum amplitude response of the primary system, corresponding to H_∞ optimization.

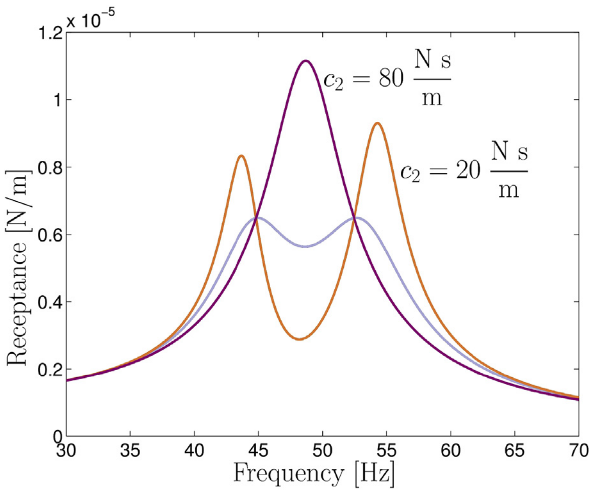

Fig. 1. Illustration of Den Hartog's equal-peak method, $\omega_1 = 50$ Hz, $m_1 = 10$ kg, $\eta = 0.05$. Equal peaks are obtained for $k_2 = 4.48e4$ N/m and $c_2 = 39.98$ N s/m.