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Non-material finite element modelling of large vibrations of axially moving strings and beams

Yury Vetyukov

Vienna University of Technology, Institute of Mechanics and Mechatronics, Getreidemarkt 9, A-1060 Vienna, Austria.

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In memory of my teacher Prof. Vladimir Eliseev

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ABSTRACT

We present a new mathematical model for the dynamics of a beam or a string, which moves in a given axial direction across a particular domain. Large in-plane vibrations are coupled with the gross axial motion, and a Lagrangian (material) form of the equations of structural mechanics becomes inefficient. The proposed mixed Eulerian-Lagrangian description features mechanical fields as functions of a spatial coordinate in the axial direction. The material travels across a finite element mesh, and the boundary conditions are applied in fixed nodes. Beginning with the variational equation of virtual work in its material form, we analytically derive the Lagrange's equations of motion of the second kind for the considered case of a discretized non-material control domain and for geometrically exact kinematics. The dynamic analysis is straightforward as soon as the strain and the kinetic energies of the control domain are available. In numerical simulations we demonstrate the rapid mesh convergence of the model, the effect of the bending stiffness and the dynamic instability when the axial velocity gets high. We also show correspondence to the results of fully Lagrangian benchmark solutions.

1. Introduction

Axially moving strings or beams have long attracted the attention of mechanical engineers. Practical relevance, technical difficulties in maintaining the desired regime of motion, non-trivial and even sometimes counter-intuitive behavior are coupled with challenges, intrinsic for the theoretical investigation of such systems. Corresponding mathematical models traditionally feature a spatial (or Eulerian) description, see review papers by Chen [1] and Marynowski and Kapitaniak [2]. Considering unknowns displacements, forces, moments, etc. as functions of a fixed coordinate in the axial direction simplifies the analysis, as the problem needs then to be solved in a fixed control domain and the boundary conditions are applied at fixed points. On the other hand, the basic equations of structural mechanics are available in the Lagrangian form, when the mechanical fields are observed in material points. As shown below in section 2, transforming the equations to the spatial form is simple in the linear case, when the relation between the material and the spatial coordinates is

E-mail address: yury.vetyukov@tuwien.ac.at (Y. Vetyukov).

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known in advance. In nonlinear problems, many authors adopt certain kinematical simplifications and derive the mathematical model in the spatial frame from scratch [3,4], or rely upon other known equations from the literature. Thus, the models of Wickert [5] and Mote [6] are often applied, see e.g. Refs. [7–10]. While numerous fascinating and practically important results were obtained in this way, accurate modelling of large vibrations of axially moving structures requires exact treatment of geometrically nonlinear effects both in the elastic response as well as in the inertial properties with an established structural mechanics theory in the background. Despite growing interest of researchers [11–16], developing efficient and reliable techniques of transforming the general equations of motion to a new spatial form remains a challenging problem.

In computational mechanics of flexible solids, dynamical modelling of a discretized system is convenient with Lagrange's equations of motion of the second kind. For a given finite element (or global Ritz) approximation one simply integrates the total strain energy and kinetic energy as functions of generalized coordinates and velocities, and derivatives of these functions constitute the equations. The approach is particularly straightforward when combined with the material description of the kinematics of deformation and motion. Considering mechanical fields as functions of coordinates in the reference configuration is advantageous because an elastic body keeps "memory" of its undeformed state. Moreover, dealing with the same material volume ensures the validity of Lagrange's equations of motion. On the contrary, in fluid mechanics it becomes more efficient to observe the processes at given points in space. But the needs of modern computational mechanics go beyond the simple ideas of the past, and the so-called Arbitrary Lagrangian-Eulerian (ALE) formulations are nowadays increasingly used for problems of fluid-structure interaction, material forming processes, etc. This family of methods features control volumes, which are moving in a problem-oriented manner relative to both the spatial actual state as well as the reference configuration [17]. In their traditional form, ALE methods imply accounting for the advection of material in the local forms of the constitutive and balance equations. An interesting application to structural mechanics, which is close to the ideas of the present study, has been presented by Hong and Ren in Ref. [18] and by Yang et al. in Ref. [19]. The authors of the mentioned papers make use of a redundant set of degrees of freedom in a finite element model with additional constraints. This flexible formulation is, however, potentially more complicated and less efficient than the present one as a differential-algebraic system of equations needs to be integrated over time.

While exploiting Lagrange's equations of motion is efficient for large vibrations of structural members [20], dynamics of flexible rotors [21], etc., they cannot be directly applied to axially moving structures owing to the non-material control volume under consideration. By proper handling the flow of momentum across the boundaries of an open system, one may extend Hamilton's principle to the present case of a non-material volume [22,23]. Modelling axially moving structures, researchers frequently make use of these equations, see e.g. Refs. [24,25]. Nevertheless, equations of motion for open systems obtained with the methods of Newtonian mechanics and extended Hamilton's principle may not always be identical, see comparisons and discussions in Stangl et al. [26] as well as in Chapter 5 and in Appendix E of Païdoussis [27].

Comprehensive studies on extending Lagrange's equations of motion to open systems have been reported by Irschik and Holl [28,29], who arrived at the traditional form of equations augmented by certain integral terms over the boundaries with material flow. Taking into account the high level of abstraction of the mentioned results, in the present paper we seek complete clarity and transparency for the considered particular class of problems of axially moving continua with kinematic boundary conditions. The scientifically new results of this contribution are the following.

1. We continue the research on a novel mixed Eulerian-Lagrangian kinematic description, which allows for the geometrically exact treatment of large deformations of axially moving multi-dimensional structures. Earlier [30,31] the approach was proven to be efficient for large quasistatic deformations with no effect of inertia. An extension to dynamics is now exposed for the first time.
2. The equation of virtual work (D'Alembert's principle in dynamics), which is originally available in the material form, is transformed to the present spatial description. The new form can be applied per se for both numerical simulations as well as constructing analytical solutions, and is essentially different from the ALE methods discussed above.
3. Introducing a Galerkin-Ritz approximation of the unknown displacement field over the spatial coordinate in the non-material control domain and assuming kinematic boundary conditions, we further transform the variational equality to Lagrange's equations of motion of the second kind for the generalized degrees of freedom.
4. On an example problem with large vibrations we demonstrate the rapid mesh convergence of a simple finite element scheme and validate the solutions against available benchmark results of fully Lagrangian simulations.

In the following three sections of the paper we establish the basic notions and present the approach on the simple example of one-dimensional axial vibrations of a rod. Similar kinematic relations can be found in earlier works [14,16], in which the analysis is focused on the differential equations of motion, and not on the variational equations. The actual mechanics of transverse vibrations of axially moving strings and beams is discussed in section 5. Although the range of possible configurations is restricted by the condition that the axial coordinate must increase monotonously along the material line (see discussion after Eq (43)), large vibration amplitudes are treated geometrically exactly. In section 6 we present extensive numerical simulations, verifications and tests.