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Vibration of a string against multiple spring-mass-damper stoppers

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When a building sways due to strong wind or an earthquake, the elevator rope can undergo resonance, resulting in collision with the hoist-way wall. In this study, a hard stopper and a soft stopper comprised of a spring-mass-damper system installed along the hoist-way wall were considered to prevent the string from undergoing excessive vibrations. The collision of the string with multiple hard stoppers and multiple spring-mass-damper stoppers was investigated using an analytical method. The result revealed new formulas and computational algorithms that are suitable for simulating the vibration of the string against multiple stoppers. The numerical results show that the spring-mass-damper stopper is more effective in suppressing the vibrations of the string and reducing structural failure. The proposed algorithms were shown to be efficient to simulate the motion of the string against a vibration stopper.

1. Introduction

As building heights increase, the lengths of elevator ropes also increase. High-rise buildings have low natural frequencies compared to conventional buildings. Therefore, when a building sways due to strong wind or an earthquake, an elevator rope can undergo resonance and thus collide with the hoist-way wall. To overcome this problem, suppressors have been installed in buildings in Japan. Kimura and Nakagawa [1] used the finite difference method to analyze the elevator rope when a vibration suppressor is present, and they found that the maximum rope deflection reduces as the number of suppressors increases. The vibration suppressor was modeled using a one-degree-of-freedom model consisting of a spring, a mass, and a damper. Kimura [2–4] used the finite difference method to analyze the free vibrations of an elevator rope with multiple vibration suppressors.

The simplest continuous system in vibration analysis is a flexible string and the vibration of a uniform string can be easily described analytically [5,6]. However, it is not possible to obtain an analytical solution if the vibration of the string causes a collision with the surrounding structures. Many researchers have studied the free vibration of a string against a rigid obstacle. Frontini and Gotusso [7] used a discrete model to analyze the motion of a string against an obstacle. Cabannes [8] presented algorithms to compute the motion of a string encountering diverse two-dimensional obstacles. Han and Grosenbaugh [9] carried out a non-linear free vibration analysis of a cable against a straight obstacle. Ahn [10] studied a vibrating string with dynamic frictionless impact.

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Actually, the use of a collision model coupled with structural vibrations is a classical research topic. The so-called coefficient of restitution (CoR) which is the ratio of the pre- and post-impact velocities plays a major role in describing the collision. Fathi and Popplewell [11] suggested numerical procedures for the peak response of an elastic beam colliding with a resilient stop. Wang and Kim [12] developed a method utilizing impulse response functions to obtain the impact problem of a cantilever beam colliding with a rod. Wagg et al. [13] carried out experiments on a vibro-impacting cantilever beam. Metallidis and Natsiavas [14] investigated the periodic response of a deformable rod with clearance and a motion-limiting stop. Wagg and Bishop [15] used impact condition at the stop. However, in their analysis, the number of modes needs to be equal to the number of points that are considered on the beam including the stop. Fegelman and Grosh [16] investigated a vibro-impacting problem of a beam contacting a linear spring. Krishnaswamy and Smith [17] studied the vibration of a string colliding with rigid obstacles. They proposed a method based on digital waveguides and finite-difference method to simulate an ideal string colliding with inelastic obstacles. Wagg [18] used the collocation method to obtain the velocities immediately after impact. Wagg [19] modeled the impact induced vibration based on the CoR.

Nevertheless, the vibration analysis of a structure against a rigid or an elastic obstacle is not an easy task. Recently, Vyasarayani [20] proposed a new method based on the unit impulse response and modal equations. Using this method, the relation among the modal velocities before and after impact can be determined, avoiding the need to integrate the equations of motion during impact. Vyasarayani [20] indicated that the following conditions should be satisfied during impact. First, the configuration of the system should not change during impact according to a conventional rigid-body CoR approach. Second, the velocity field of the structure changes only at the point of impact according to the CoR method, setting up a non-smooth velocity field after impact. Despite the success of modeling short-duration impacts in flexible multibody systems using Vyasarayani's [20] CoR approach, the extension of CoR-based modeling to structural systems has received little attention.

In this study, the work by Vyasarayani [20] is extended to string vibrations with multiple soft stoppers consisting of a spring-mass-damper system. Also, computational algorithms are also simplified to be easily implemented on a computer program. The numerical results show that the proposed algorithms can efficiently simulate the motion of a string with vibration stoppers.

2. Vibrations of a uniform string

The equation of motion for a uniform string is described using

$$\rho \frac{\partial^2 u(x,t)}{\partial t^2} + T_s \frac{\partial^2 u(x,t)}{\partial x^2} = f(x,t) \tag{1}$$

where ρ is the mass per unit length, $u(x,t)$ is the string displacement, T_s is the tension of the string, and $f(x,t)$ is the distributed force acting on the string. For a uniform string with both ends fixed, the natural frequencies and natural modes can be obtained as

$$\omega_i = i\pi\sqrt{\frac{T_s}{\rho L^2}}, \tag{2a}$$

$$\mathbf{\Phi}_i(x) = \sqrt{2}\sin\frac{i\pi x}{L}, \quad i = 1, 2, \ldots \tag{2b}$$

where L is the length of the string.

If we express the displacement by using n eigenfunctions and modal coordinates, we can then write.

$$u(x,t) = \sum_{i=1}^{n} \Phi_i(x) q_i(t) = \mathbf{\Phi q} \tag{3}$$

where $\mathbf{\Phi} = [\Phi_1\ \Phi_2 \ldots \Phi_n]$ is the matrix consisting of eigenfunctions and $\mathbf{q} = [q_1\ \ q_2\ \ \ldots\ \ q_n]^{\mathrm{T}}$ is the modal displacement vector. The matrix-vector notation is used to avoid summation symbols. Using Eq. (3), the modal equations of motion assuming the modal damping term can then be written as

$$\ddot{\mathbf{q}} + 2\mathbf{Z\Omega}\dot{\mathbf{q}} + \mathbf{\Lambda q} = \mathbf{Q} \tag{4}$$

where $\mathbf{Z} = \mathrm{diag}([\zeta_1\ \ \zeta_2\ \ \ldots\ \ \zeta_n])$, $\mathbf{\Omega} = \mathrm{diag}([\omega_1\ \ \omega_2\ \ \ldots\ \ \omega_n])$, $\mathbf{\Lambda} = \mathrm{diag}([\omega_1^2\ \ \omega_2^2\ \ \ldots\ \ \omega_n^2])$, and $\mathbf{Q} = \int_0^L \mathbf{\Phi}^{\mathrm{T}}(x) f(x,t) dx/\rho L$, in which ζ_i is the damping factor for the ith natural mode. The modal equation of motion, Eq. (4), can be transformed into the state-space equation. Then, we can obtain

$$\dot{\mathbf{z}} = \mathbf{Az} + \mathbf{BQ} \tag{5}$$

where $\mathbf{z} = [\mathbf{q}^{\mathrm{T}}\ \ \dot{\mathbf{q}}^{\mathrm{T}}]^{\mathrm{T}}$ and