Journal of Sound and Vibration 413 (2018) 101–119

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Effect of thermal stresses on frequency band structures of elastic metamaterial plates

Ying Wu [a], Kaiping Yu [a,*], Linyun Yang [a], Rui Zhao [a], Xiaotian Shi [b], Kuo Tian [c]

[a] Department of Astronautic Science and Mechanics, Harbin Institute of Technology, Harbin, Heilongjiang Province 150001, China
[b] Department of Aeronautics and Astronautics, University of Washington, Seattle, WA 98195, USA
[c] Department of Engineering Mechanics, Dalian University of Technology, Dalian, Liaoning Province 116024, China

ARTICLE INFO

Article history:
Received 6 June 2017
Received in revised form 3 October 2017
Accepted 7 October 2017
Available online 16 November 2017

Keywords:
Nonlinear effect
Band structure
Metamaterial plate
Thermal stresses
Sensitivity

ABSTRACT

We investigate the effect of thermal stresses on the band structure of elastic metamaterial plates by developing a useful finite-element based method. The thermal field is assumed to be uniform throughout the whole plate. Specifically, we find that the stiffness matrix of plate element is comprised of elastic and thermal stresses parts, which can be regarded as a linear function of temperature difference. We additionally demonstrate that the relative magnitudes between elastic properties and thermal stresses will lead to nonlinear effects on frequency band structures based on two different types of metamaterial plates made of single and double inclusions of square plates, respectively. Then, we validate the proposed approach by comparing the band structures with the frequency response curves obtained in finite periodic structures. We conduct sensitivity analysis and discuss in-depth the sensitivities of band structures with respect to temperature difference to quantitatively investigate the effect of thermal stresses on each band. In addition, the coupled effects of thermal stresses and temperature-dependent material properties on the band structure of Aluminum/silicone rubber plate have also been discussed. The proposed method and new findings in this paper extends the ability of existing metamaterial plates by enabling tunability over a wide range of frequencies in thermal environments.

1. Introduction

In recent decades, phononic crystals and acoustic metamaterials have captured much attention from the scientific community due to their superb dynamic properties and potential applications in vibration attenuation, acoustic cloaking, acoustic hyperlens, etc. [1,2]. More recently, an elastic metamaterial plate [3–5], defined as a thin structure with inclusions periodically arranged, has also been explored with the view from a wide range of engineering applications.

Most elastic metamaterial plates [3–9] proposed to date are characterized to operate at fixed frequency ranges, limiting the possible applications. To improve the functionality of metamaterials, it is desirable to actively tune the band structures [10] within the same fabricated sample. Among various types of active methods (such as piezoelectric materials [11–17]), temperature-based technique, which exploits temperature-dependent material to fabricate metamaterial or a heterogeneous thermal expansion of constituent components, is another choice. Several attempts have been made to investigate tunability of

* Corresponding author.
E-mail address: yukp@hit.edu.cn (K. Yu).

https://doi.org/10.1016/j.jsv.2017.10.014
0022-460X/

metamaterial influenced by temperature. Jim et al. [18] reported a tunable ferroelectric ($Ba_{0.7}Sr_{0.3}$-TiO_3)/epoxy phononic crystal and demonstrated that tuning temperature will result in a significant change of the band structure. Later, Cheng et al. [19], Yao et al. [20], Bian et al. [21], and Hu et al. [22] also investigated the temperature tunable phononic crystals made of $Ba_{0.7}Sr_{0.3}TiO_3$. However, the study of temperature effects on the band structure of phononic crystal mainly focuses on temperature-dependent materials such as ferroelectric, whose properties are very sensitive to the temperature around the Curie temperature. Recently, Li et al. [23] investigated the thermal tuning property of one-dimensional phononic crystal based on temperature-dependent elastic property of the interfacial adhesive layer. For thermal effect of thermal stresses, Wang et al. [24] investigated the propagation and localization of elastic waves in 1-D randomly disordered layered phononic crystals. They claimed that the cut-off frequencies of passbands and stopbands can be tuned by varying temperature. The authors [25] explored the thermoelastic wave band in 2-D and 3-D phononic crystals and proposed a generalized PWE (plane wave expansion) method based on GN (Green and Naghdi) theory. We concluded that generalized thermoelastic bands are composed of separate elastic and thermal bands.

Since the elastic metamaterial plates have the ability to block vibration and noise in a certain frequency range [26,27], it is a feasible approach to regard the structures as a kind of acoustic covering which can attenuate the high-level sound pressure on some structures, such as hypersonic flight vehicles. During the entire lifetime, hypersonic flight vehicles will suffer extremely serious load conditions such as vibration, heat, noise, acceleration, and low gravity [28,29]. The characteristic of hypersonic flight vehicle structures will be modified owing to the intense thermal stresses, which results from aerodynamic heating during the process of flying at high speeds. Therefore, once elastic metamaterial plates are subjected to unignorable temperature change, a reliable model to accurately predict the temperature-dependent bandgap of elastic metamaterial plates is of high demand.

Generally, to characterize the dynamic properties of such metamaterial plates, several effective methods have been adopted, such as TFM [30] (transfer matrix method), PWE [31], FDTD [32] (finite domain time difference), SEM [33] (spectral element method). However, neither PWE nor FDTD has the ability to model the elastic metamaterial with thermal stresses. Nouh et al. [34,35] developed a FEM (finite element method) model based on Kirchhoff's plate theory to predict the wave propagation in metamaterial plates with periodic local resonances. Such method enjoys fast convergence as well as saving much time. However, since Kirchhoff's theory does not consider the independent shearing along thickness, it is only suitable for thin plate. However, to the best of our knowledge, there is little literature considering the effects of thermal stresses on band structures of elastic metamaterial plates. The primary precondition of calculating the band structure of metamaterial plates is Bloch-Floquet theorem which implies that the periodic structures should theoretically possess infinite periods [2]. But it is well known that such infinity of periodic structures cannot be realized in practice. For a metamaterial plate with finite size embedded in the thermal environment, thermal stresses will occur if the expansion or compression cannot happen freely because of boundary and internal constraints. Consequently, the effects of thermal stresses cannot be ignored since the magnitudes of thermal stresses in some regions are comparable to the elastic constants [36,37].

In this paper, we will investigate the effect of thermal stresses on dispersive curves of elastic metamaterial plates in order to achieve the possibility to tune the frequency band structure in thermal environments. Aluminum/Silicone rubber and Lead/Silicone rubber/Epoxy elastic metamaterial plates are used for illustration and discussion. Based on the proposed approach, we know that the overall stiffness matrix is composed of elastic and thermal stress parts. And the thermal stress part is a linear function of temperature difference. We demonstrate that both values and shapes of band structures can be tuned remarkably by changing temperature. Additionally, the nonlinear effect of thermal stresses on each band is not identical which can be further interpreted by sensitive analysis. The band structures are validated by comparing with frequency response curves obtained by COMSOL Multiphysics. There is a good agreement between our predictions and the frequency response curves.

2. Geometrical configurations of elastic metamaterial plates

In this study, we investigate frequency band structures of two different types of plates under the influence of thermal stresses: Aluminum/Silicone rubber (Al-Si) and Lead/Silicone rubber/Epoxy (Ld-Si-Ex) plates. These Al-Si and Ld-Si-Ex metamaterial plates embed relatively heavy and stiff masses in light and soft matrix by using single- and double-inclusion configurations, respectively (see Fig. 1 and Table 1 for their geometrical configurations and material properties [25]). Note that the embedded heavy masses are not attached to the surface of the soft matrix but inserted into the plate through the open cut. The difference between these two configurations is that the single-inclusion configuration simulates a phononic crystal structure, while the double-inclusion one mimics a locally resonant metamaterial. The thickness of the plates is $h = 0.002$ m. Without loss of generality, both matrix lattices and the inclusions are in the shape of a square with the corresponding lattice constant $a = 0.02$ m.

3. Thermal stresses distributions in elastic metamaterial plates

In order to obtain the distribution of thermal stresses in the plate, a four-node plane stress element is developed with the following assumptions [38–41]: (1) The body is small in the z direction in comparison to the other two directions. Thickness h is generally less than or equal to one-tenth of the smallest dimension in the xy plane. (2) The body is subjected to loading in the xy plane. (3) The material of the body is linearly elastic, isotropic, and homogenous.