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Dynamic analysis of geometrically non-linear three-dimensional beams under moving mass

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ABSTRACT

In this paper, we present a coupled dynamic analysis of a moving particle on a deformable three-dimensional frame. The presented numerical model is capable of considering arbitrary curved and twisted initial geometry of the beam and takes into account geometric non-linearity of the structure. Coupled with dynamic equations of the structure, the equations of moving particle are solved. The moving particle represents the dynamic load and varies the mass distribution of the structure and at the same time its path is adapting due to deformability of the structure. A coupled geometrically non-linear behaviour of beam and particle is studied. The equation of motion of the particle is added to the system of the beam dynamic equations and an additional unknown representing the coordinate of the curvilinear path of the particle is introduced. The specially designed finite-element formulation of the three-dimensional beam based on the weak form of consistency conditions is employed where only the boundary conditions are affected by the contact forces.

1. Introduction

Deformable three-dimensional structure under moving body represents a complex and very demanding problem. Due to complexity of the problem many partial and simplified models can be found in literature. Many authors study separate parts of this otherwise strongly coupled problem. The structure is usually modelled with plane beam elements or in some cases with rigid bodies connected with linear or non-linear springs. The moving body is often modelled as mass particle or rigid body focusing on structural response. Several authors base their studies of this problem on geometrically linear simply-supported Euler-Bernoulli plane beam element: [1–6]. Continuous flexible plane beam replaced by a system of rigid bars and flexible joints was the basis of the technique by Mofid and Akin [7] for determining the behaviour of beams with different boundary conditions carrying a moving mass. Several researchers studied geometrically linear Timoshenko beam subjected to traveling mass, e.g., [8–16]. Yavari et al. [17] replace the Timoshenko beam with system of rigid bars and flexible joints. Wu, Whittaker and Cartmell [18,19] expose the necessity for more general models for dynamic response of structures subjected to moving loads with motivation in improved control of mobile gantry cranes and propose several approximative methods for considering these problems using standard finite-element packages. For similar reasons the methodology of Saleeb and Kumar [20] addressed the applicability of standard finite element packages for bridge-vehicle interaction problems.

Recent publications study various interesting phenomena and influential parameters in moving mass transition over a beam-like structure. Ahmadi and Nikkhoo [21] and Zhao et al. [22] analyze the effect of varying cross-section. Bilello et al. [23] propose a correction procedure for better estimation of response in high-frequency domain. Influence of intermediate supports is ana-

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lyzed by DeSalvo et al. [1] and Ebrahimi et al. [24]. Dyniewicz [9] applies the velocity-based space-time finite element method to moving mass problem. An interesting approach of Hasheminejad and Rafsanjani [25] employs the two dimensional elasticity model to avoid the problems with the thickness of the beam. Lin and Lee [26] study in-plane vibration of curved beams under moving mass. Baeza and Ouyang [27] analyze the vibration caused by a moving oscillator on a truss structure composed of Timoshenko beams. Mamand and Kargarnovin [28,29] consider the geometric non-linearity and beam inclination in their analysis. Initially inclined beam is also studied by Wu [30]. Dynamic instability and resonance conditions using harmonic balance method were presented by Pirmoradian et al. [31]. Response of the Euler-Bernoulli beam with moving mass under horizontal support excitation is investigated by Zarfam et al. [32]. Andersen et al. [33] model the influence of the moving load on ground vibration.

Most of the above mentioned publications are limited to vibration analysis of planar geometrically linear beams. The primary purpose of this work is to present a geometrically non-linear three-dimensional beam formulation under moving mass. More sophisticated three-dimensional models of beam-like structures introduce computationally more demanding systems of differential equations. Among them the *Cosserat theory of rods* [34], is widely used. The numerical implementation of the model is usually attributed to Reissner [35] and Simo [36], where it is also called *the geometrically exact beam*. Dynamic analysis of three-dimensional Cosserat rods has been a subject of wide research, see, e.g. [37–40], among many others who contributed to the topic. The crucial step in developing numerical methods for dynamic analysis of three-dimensional beams represents the discretization with respect to time. We should be aware that the dynamics of a three-dimensional frame is often described by a system of stiff differential equations. Additionally, we need to consider properly the three-dimensional rotations which represent a demanding mathematical structure. Therefore, their behaviour in space and time is non-trivial, which requires a special treatment in numerical formulations. Despite the well known and widely documented difficulties with finite spatial rotations they are often taken to be the primary variables in three-dimensional beam formulations. Authors use various parameterizations of finite rotations. The chosen parameterization of rotations directly affect the overall numerical procedure, see, e.g. Ref. [41]. Interesting, but surprisingly rarely used in beam formulations, is the parametrization of rotations with four-dimensional quaternions. In contrast to standard parameterizations of rotations with rotational vectors, quaternions are singularity-free and computationally more effective. Finite elements employed here are based on rotational quaternions as the only quantity that describes the rotations of the cross-sections of the beam. In time discretization we follow the pioneering work of Simo and Vu-Quoc [42] where the time discretization of rotations was treated consistently. The adaptation of these ideas for the time integration scheme on rotational quaternions as presented by Zupan et al. [43] is employed here.

Our approach in finite-element formulation is very specific in order to make possible numerically efficient coupling of the finite element with the equations of a moving body. It is essentially a collocation-type of discretization where the equilibrium conditions for forces and moments are required to be satisfied in strong (integrated) form at the two boundary points, while the field equilibrium equations in the differential (weak) form are satisfied point-wise at a specified set of interior collocation points. This approach preserves the symmetry of equations with respect to the length of the beam after the discretization and otherwise algebraic boundary conditions are transformed into differential ones. Such procedure avoids the necessity of solving mixed algebraic differential equations which could be computationally demanding. The influence of the moving body in such formulation affects only the strong boundary equations. It is treated in accord with the theory of distributions. Numerical tests show no problems in the transition over pair of finite elements.

2. Model of a three-dimensional beam

A three-dimensional beam is described by the line of centroids and the family of rigid cross-sections, Fig. 1. The centroidal line at an arbitrary time $t \geq 0$ is defined by position vector function $\mathbf{r}(x, t)$, where $x \in [0, L]$ is the arc-length parameter of line of centroids at $t = 0$. The orientation of each cross-section $\mathcal{A}(x, t)$ is defined by local orthonormal bases $\mathcal{B}_G(x, t) = \{\mathbf{G}_1(x, t), \mathbf{G}_2(x, t), \mathbf{G}_3(x, t)\}$. After introducing a global fixed orthonormal basis $\mathcal{B}_g = \{\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3\}$ any particular cross-section can be uniquely described by rotation between the fixed basis and the local one.

Among several possible ways of describing and parametrizing rotations we here employ the rotational quaternions. Using the algebra of quaternions the relation between the moving and the fixed basis can be written as

$$\mathbf{G}_i(x, t) = \widehat{\mathbf{q}}(x, t) \circ \mathbf{g}_i \circ \widehat{\mathbf{q}}^*(x, t), \quad i = 1, 2, 3, \tag{1}$$

where ($\circ$) denotes the quaternion product, $\widehat{\mathbf{q}}$ is the rotational quaternion, and $\widehat{\mathbf{q}}^*$ its conjugate. The space of quaternions is a four-dimensional vector space over real numbers. Rotational quaternions are expressed as

$$\widehat{\mathbf{q}} = \cos\frac{\vartheta}{2} + \sin\frac{\vartheta}{2}\,\mathbf{i} = \begin{bmatrix} \cos\frac{\vartheta}{2} \\ \sin\frac{\vartheta}{2}\,\mathbf{i} \end{bmatrix}, \tag{2}$$

where ϑ is the angle of rotation and $\mathbf{i}$ is the unit vector on the axis of rotation. Vectors in IR^3 are treated as elements of the space of quaternions, where $\mathbf{v} \equiv \widehat{\mathbf{v}} = \begin{bmatrix} 0 & v_1 & v_2 & v_3 \end{bmatrix}^{\mathrm{T}}$. The hat over the symbol will be omitted when the first component equals zero since the appropriate size of a vector will always be evident from the context. A comprehensive presentation of the quaternion