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Non-planar vibrations of slightly curved pipes conveying fluid in simple and combination parametric resonances

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The paper summarises the experimental investigations and numerical simulations of non-planar parametric vibrations of a statically deformed pipe. Underpinning the theoretical analysis is a 3D dynamic model of curved pipe. The pipe motion is governed by four non-linear partial differential equations with periodically varying coefficients. The Galerkin method was applied, the shape function being that governing the beam's natural vibrations. Experiments were conducted in the range of simple and combination parametric resonances, evidencing the possibility of in-plane and out-of-plane vibrations as well as fully non-planar vibrations in the combination resonance range. It is demonstrated that sub-harmonic and quasi-periodic vibrations are likely to be excited. The method suggested allows the spatial modes to be determined basing on results registered at selected points in the pipe. Results are summarised in the form of time histories, phase trajectory plots and spectral diagrams. Dedicated video materials give us a better insight into the investigated phenomena.

1. Introduction

Pipes conveying fluids are widely used in various sectors of industry. Under certain conditions, however, the interactions between the pipe and fluid flowing with varying velocity lead to dangerous dynamic behaviours, and sometimes the entire hydraulic installation gets damaged. On the other hand the involved phenomena have become an interesting research field and a subject of extensive studies. Development of models of such systems alongside the improved calculation powers permits a more accurate description of even very complicated systems. In comparison to this, a much smaller amount of experimental works is felt. On one hand experimental tests enable the model validation, on the other-they offer us a better insight into the processes which have not been thoroughly researched before.

The works by Paidoussis [1,2] and Ibrahim [3] provide an overview of studies investigating the dynamic behaviour of pipes conveying fluids. The problem has been investigated from various perspectives, numerous researchers investigated straight and curved pipes, varying pipe supports (one- or two-ended supports, rigid or elastic), the pipe behaviour during the flow with and without velocity pulsation. Underpinning the computational models have been the string models as well as diverse

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beam and shell models. The models used are mostly 2D models, though 3D models better capturing the investigating phenomena are now often considered.

This study is a continuation of the work [4]. In that article a 3D model of a pipe was proposed, used to investigate the vibrations of statically deformed elastic pipes. The introductory section of that article contains references to other works whose research efforts focused on modelling and investigating the behaviours of curved pipes.

Relatively little published literature is available on parametric vibrations of 3D models, especially for pipes fixed at both ends. Most studies investigate the behaviour of cantilevered pipes. Nonlinear models of pipes with a free end were considered by Bajaj and Sethna [5]. Copeland and Moon [6] studied pipes with an additional mass, the works by Steindl and Troger [7,8] focused on flows with motion constraints in the form of various arrays of springs. In the three-part work [9–11], equations were derived that govern 3D motions of pipes under steady flow conditions and the pipe dynamics was investigated for several variants of elastic pipe supports and for a pipe with an end mass attached. This model was verified experimentally by Ghayesh et al. [12,13]. In studies on 3D pipes vibration, the authors often analyze the shape of vibration modes and the motion trajectories of selected points of pipe.

Analyses of vibrations of pipes fixed at both ends mostly rely on 2D models. For flows with velocity pulsation, such models well capture the resonance phenomena, allowing those ranges of system parameters to be identified in which parametric vibrations are excited. In the case of combination resonance, however, such simplified models prove inadequate for a detailed description of the occurring phenomena.

In the case of non-planar models of pipes fixed at both ends, the main focus is on stability and eigenvalue problems. Chen [14] analysed a linear model of a curved pipe, demonstrating that the stability loss in for the pipe model with a non-deformable axis occurs when the critical velocity is exceeded, similar to straight pipe. Hill and Davis [15] assumed that the pipe axis is deformable and showed that a curved pipe is not be subjected to in-plane and out-of-plane buckling. Misra et al. [16,17] compared various models of curved pipes, concluding that the most adequate model should be that taking into account deformability of the pipe axis. Jung and Chung [18] employed the Galerkin method to determine the natural frequencies of in-plane and out-of-plane vibrations of a nonlinear model of a semi-circular pipe conveying fluid. Taking into account geometric nonlinearity and various types of physical nonlinearities, they demonstrated that the nonlinear model does not buckle even at high flow velocities.

Few studies investigate the parametric resonance for 3D models of curved pipes clamped at both ends. For example, Jung et al. [19] explored a nonlinear model of semi-circular pipe with pulsating flows. Recalling the Floequet method, they determined the simple resonance ranges for the first and second modes and for their combination resonance. Nakamura et al. [20] and Yamashita et al. [21] undertook theoretical and experimental investigations of a circular pipe with pulsating flow to highlight the interactions between in-plane and out-of-plane vibrations, stating excitation of out-of-plane parametric resonance. Ni et al. [22] developed a nonlinear model of a curved pipe, determined natural frequencies of its in-plane and out-of-plane vibrations and analysed the effects of flow velocity, mass and internal damping on the ranges of parametric resonance. They compare predicted and experimental data, demonstrating the excitation of the out-of-plane parametric resonance. Lu et al. [23] investigated the principal parametric resonance phenomena for a model of a curved pipe with geometrical non-linearities, demonstrating the coupling of in-plane and out-of-plane vibrations.

This study summarises results of theoretical and experimental research of slightly curved, elastic pipe. Numerical simulations use a 3D nonlinear model of the system, proposed in Ref. [4], describing the transverse vibrations in two perpendicular directions as well as axial and torsional vibrations. The Gallerkin method is recalled and the approximate solution takes into account eight modes. The main focus is on 3D modes in the range of simple and combination parametric resonance. To visualise the pipe shape during resonance vibrations, the experimental data have been post-processed accordingly.

2. Governing equation

In the paper [4] differential equations of motions are derived which govern the dynamic behaviour of a statically deformed pipe conveying pulsating fluid. In Appendix A, the most important assumptions and short explanations of the studied equations are given. The fluid is assumed to be incompressible, the flow velocity remains unchanged along the pipe length and is taken to be constant over the pipe's cross section. Static deformations of an elastic pipe, mostly due to gravity forces, are taken into account.

The model of the slightly curved pipe is shown in Fig. 1. The state of equilibrium is defined by the system of the first-order differential equations:

$$N'_{10} = k_0 N_{20} - m_f \lambda_0 U_{f0}^2 - m_p g \sin \chi_0 \tag{1}$$

$$N'_{20} = -k_0\left(N_{10} + S_0 - p_0 A_f - m_f U_{f0}^2\right) - mg \cos \chi_0 \tag{2}$$

$$\chi'_0 = k_0 \tag{3}$$

$$k'_0 = -N_{20}/EI_3 \tag{4}$$