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Dark solitons, modulation instability and breathers in a chain of weakly nonlinear oscillators with cyclic symmetry

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ABSTRACT

In the aerospace industry the trend for light-weight structures and the resulting complex dynamic behaviours currently challenge vibration engineers. In many cases, these light-weight structures deviate from linear behaviour, and complex nonlinear phenomena can be expected. We consider a cyclically symmetric system of coupled weakly nonlinear undamped oscillators that could be considered a minimal model for different cyclic and symmetric aerospace structures experiencing large deformations. The focus is on localised vibrations that arise from wave envelope modulation of travelling waves. For the defocussing parameter range of the approximative nonlinear evolution equation, we show the possible existence of dark solitons and discuss their characteristics. For the focussing parameter range, we characterise modulation instability and illustrate corresponding nonlinear breather dynamics. Furthermore, we show that for stronger nonlinearity or randomness in initial conditions, transient breather-type dynamics and decay into bright solitons appear. The findings suggest that significant vibration localisation may arise due to mechanisms of nonlinear modulation dynamics.

1. Introduction

Localisation of vibrations has received considerable attention from the structural dynamics engineering community over the last three decades (see e.g. papers [1–8] and references therein). In a linear framework, localisation may arise due to imperfections in the manufacturing process that result in a slightly disordered and inhomogeneous system. The localisation of vibrations has particular relevance in the aerospace industry, where e.g. bladed-disks of turbo-machines, reflectors, and antennas are usually composed of ideally periodic structures. Linear localisation due to disorder in general was first observed in solid state physics [9], and this community usually refers to the phenomenon as Anderson Localisation. In the aerospace and turbo-machinery community, localisation due to imperfections is usually referred to as a mistuning [10,11], when viewed from a spectral framework. Mistuning plays a significant role in system dynamics due to its importance in mechanical vibrations, fatigue, and even aerodynamics [12].

In the case of turbo-machinery applications, more and more so-called blisks are used in current aero-engine designs. A

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blisk is a blade-integrated disk, i.e. a rotating component in which the traditional separation between a disk and attached blades is overcome enabling the whole structure to be formed monolithically. Blisks thus do not have any internal mechanical joints, with their corresponding friction mechanisms, and they are usually considered undamped structures [13]. Moreover, the dynamical response of blisks under operation is often thought to be beyond the traditional range of applicability for linear models due to the effect of geometric nonlinearity for large vibration amplitudes.

A number of publications (e.g. Refs. [4,7,14–20]) have already shown that perfectly symmetric or perfectly periodic structures may localise vibrations in the nonlinear regime due to: (i) the dependence of mode shapes on displacement amplitude, which is usually referred to as non-similar modes; and (ii) through bifurcations of main normal mode branches. The role and effects of such localised solutions are still under study within the vibration engineering community, and also the relevance of geometric nonlinearities when manufacturing variability plays any role is a question of active research [21].

The emergence of travelling wave vibration states in aerospace structures has already been studied extensively, e.g. due to Coriolis effect [22] or aeroelastic excitation [12]. However, there is also a vast body of knowledge and literature on the weakly nonlinear dynamics of nonlinear travelling waves from the physics community that does not seem to be reflected widely in the present context [23,24]. Especially in the optics community the study of modulation instability has lead to substantial progress in understanding and influencing the systems under study [25]. The aim of this research is to investigate the mechanisms of vibration localisation that may arise due to modulation of travelling waves. We therefore study the stability of plane wave vibration states and the subsequent nonlinear evolution here with a view to structural dynamics engineering.

We use a highly idealised model for which we numerically study localised solutions inspired by insights from the Nonlinear Schrödinger Equation (NLSE). We find that depending on the wavelength of the travelling wave under study, the system becomes self-modulating, in the focussing parameter range, or the system can self-demodulate, in the defocussing parameter range [26]. In the self-modulating regime, the travelling nonlinear waves are linearly unstable against long-wavelength perturbations, in accordance with the general theory of modulation instability. Only in the self-demodulating regime are travelling waves linearly stable against long modulations. With a view to vibration localisation, the stable regime allows so-called dark solitons to arise, where spatially confined parts of the system do not vibrate, while the rest of the system is filled with a travelling wave. The unstable regime, as has already been reported in Ref. [27], shows bright solitonic structures, but also a complex nonlinear evolution of modulation dynamics. Depending on system parameters and initial conditions, either breather-like vibration states emerge, complex dynamics involving breathers or soliton chaos. In all cases our numerical simulations suggest that very strong vibration localisations may arise when the weakly nonlinear system properties are taken into account.

The paper is organized as follows. Section 2 introduces the analysed physical model and presents the physical framework required to deal with modulated nonlinear travelling waves. Solutions based on numerical integration are presented in Section 3. Dark solitons are introduced for defocussing range of parameters while, for the focussing parameter range, modulation instability and corresponding nonlinear breathers emergence are discussed in detail. Subsequently, Section 4 investigates the evolution of random initial conditions in the defocusing and focusing ranges. Finally, Section 5 summarises the conclusions and suggests directions for future investigations.

2. The model and solution methods

The system under study consists of N_s identical masses m cyclically connected through linear springs with constant k_c. Each mass is also connected to the ground by a linear spring k_l, and a nonlinear one k_{nl} of cubic behaviour. The corresponding configuration is depicted in Fig. 1. Physical systems similar to the one presented in Fig. 1 may be understood as a minimal model of different aerospace structures, such as space reflectors [3], disk antennas [28] and bladed-disks of aero-engines [6,7]. In the case of bladed-disk vibrations, the model presented in Fig. 1 is obtained when the Von Karman theory is applied to a system of N_s beams or plates that are cyclically coupled by massless springs, taking into account only the first

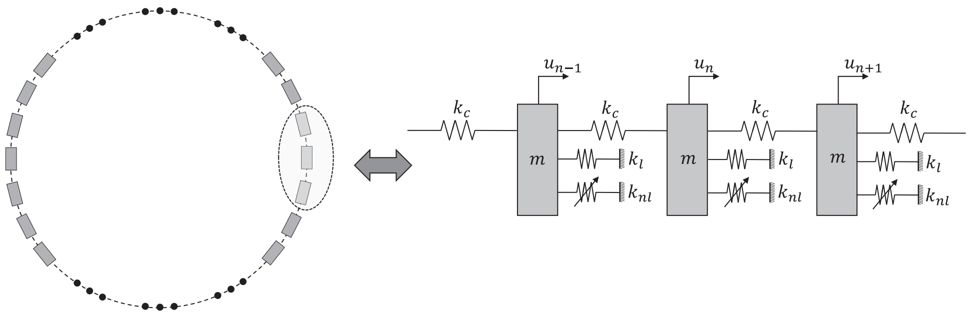

Fig. 1. Mechanical system studied. On the left hand side, an illustration of the full system, while the right hand side displays any three neighbouring degrees of freedom.

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