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A spectral-Tchebychev solution for three-dimensional dynamics of curved beams under mixed boundary conditions

Bekir Bediz *, Serdar Aksoy

Mechatronics Engineering Program, Faculty of Engineering and Natural Sciences, Sabancı University, Istanbul, 34956, Turkey

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ABSTRACT

This paper presents the application of the spectral-Tchebychev (ST) technique for solution of three-dimensional dynamics of curved beams/structures having variable and arbitrary cross-section under mixed boundary conditions. To accurately capture the vibrational behavior of curved structures, a three-dimensional (3D) solution approach is required since these structures generally exhibit coupled motions. In this study, the integral boundary value problem (IBVP) governing the dynamics of the curved structures is found using extended Hamilton's principle where the strain energy is expressed using 3D linear elasticity equation. To solve the IBVP numerically, the 3D spectral Tchebychev (3D-ST) approach is used. To evaluate the integral and derivative operations defined by the IBVP and to render the complex geometry into an equivalent straight beam with rectangular cross-section, a series of coordinate transformations are applied. To validate and assess the performance of the presented solution approach, two case studies are performed: **(i)** curved beam with rectangular cross-section, **(ii)** curved and pretwisted beam with airfoil cross-section. In both cases, the results (natural frequencies and mode shapes) are also found using a finite element (FE) solution approach. It is shown that the difference in predicted natural frequencies are less than 1%, and the mode shapes are in excellent agreement based on the modal assurance criteria (MAC) analyses; however, the presented spectral-Tchebychev solution approach significantly reduces the computational burden. Therefore, it can be concluded that the presented solution approach can capture the 3D vibrational behavior of curved beams as accurately as an FE solution, but for a fraction of the computational cost.

1. Introduction

Curved beams having variable and arbitrary cross-section find applications in many engineering and architectural areas [1]. Depending on the geometric parameters such as curvature, pretwist, width, height, and length, curved beams can represent various different structures including arches, (wind) turbine blades, springs, or stiffening elements, to name a few. Since, these curved beam structures are critical to the functional and failure characteristics of a myriad of systems, understanding and predicting the vibrational behavior of these systems is highly crucial [1–3].

A large body of literature in the last half century has been devoted to predict the vibrational behavior of curved beams/structures. Most of these studies focused on reduced order modeling approaches such as beam (one-dimensional) and plate (two-dimensional) models. Although, many researchers used beam modeling approaches (i.e., lower-order equations of

* Corresponding author.
E-mail address: bbediz@sabanciuniv.edu (B. Bediz).

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motions) to capture the vibrational behavior of curved beams, due to the coupling between the motions such as bending-axial (in-plane) motion or bending-twisting (out-of-plane) motion arising from the geometry of the structure, these beam techniques can only be used under certain simple circumstances [2,4–6]. Many modifications have been made to the classical beam (Euler-Bernoulli and Timoshenko) approaches to accurately capture the vibrational behavior of curved beams [2,6,7]. However, only lowest natural frequencies (i.e. first few natural frequencies) for curved beams with simple cross-sectional geometries (such as rectangular or circular) can be accurately captured [1].

To overcome the limitations of the reduced order models, many researchers used finite difference or finite element methods [6,8–11]. This technique enables accurate prediction of natural frequencies and mode shapes for curved structures. Furthermore, if 3D solid elements are used (that uses 3D linear elasticity theories), accurate solutions can be obtained even for curved structures having complex nonuniform geometries. However, the modeling procedure is arduous and imposes a significant computational burden.

Since the dynamics of the complex structures such as curved beams present coupled three-dimensional motions, 3D modeling approaches need to be used. These approaches are generally based on 3D linear elasticity equations [12,13] and requires numerical approaches for the solution. Therefore, series based methods such as Rayleigh-Ritz [14] or Galerkin's [6] methods are utilized. Compared to the finite element approaches, these series based approaches are computationally more efficient. However, the drawback of these techniques are **(1)** the difficulty in selecting proper basis/trial functions; since the convergence rate and accuracy highly depends on the choice of basis functions, **(2)** the need to apply a different set of basis/trial functions for each different geometry and boundary conditions.

To combine the generality of the finite elements approach with computationally efficient nature of the series based approaches, spectral element method (SEM) and quadrature element method (QEM) have been used widely in literature [15–17]. Both methods are referred as higher order finite element methods, however QEM is more versatile to apply for different problems due to the flexibility in selecting shape functions, number of node used and selection of node locations [18]. Yet, obtaining the mass and stiffness of these solution techniques are arduous and necessitates the use of special numerical algorithms.

More recently, a new series based approach that uses Tchebychev polynomials has been developed by the author for predicting the three-dimensional (rotating and stationary) dynamics of parallelepipeds and pretwisted beams [19–21]. This technique uses the integral boundary value problem (IBVP) approach; thereby eliminates the need for deriving partial differential form of the BVP and applying different basis/trial functions for each different structure and boundary conditions. Furthermore, the IBVP approach incorporates the natural boundary conditions directly to the problem and simplifies the formulation. To obtain the system matrices, recursive relations for differentiation and inner-product (integration) matrices are developed and performed exactly. For instance, in spectral-collocation methods that use Tchebychev polynomials (or any other fast converging polynomials), the equations are satisfied at each of the Gauss Lobatto (GL) points [16], whereas our solution requires that the integrals of the equations vanish with respect to all polynomials of a certain degree. The goal of this study is to advance the spectral-Tchebychev approach to enable a simple and generic solution approach that can accurately and efficiently predict the three-dimensional dynamics of curved beams having arbitrary cross-sections under unconstrained (free) and mixed boundary conditions.

This paper presents the application of the three-dimensional spectral Tchebychev (3D-ST) technique for solving the three-dimensional coupled dynamics of curved (and pretwisted) beams having arbitrary cross-sections under unconstrained (free) and mixed boundary conditions. In this approach, first, the boundary value problem of a curved beam is derived using extended Hamilton's principle. Then, necessary coordinate transformations are described to simplify the domain of the problem and to map the complex geometry into a parallelepiped geometry (straight beam with rectangular cross-section). Later, the 3D-ST technique is introduced for the solution of the transformed system of equations. To validate the presented approach and demonstrate its capabilities and computational performance, several case studies (including simple curved beams with rectangular/circular cross-sections, and curved and pretwisted beam with an airfoil cross-section) are solved and the results are compared to those found from a commercial finite elements (FE) software.

2. Derivation of the model

The geometry of a curved beam having a general cross-section and pretwist is depicted in Fig. 1. Here, $C(z)$ and $B(x, y, z)$ are the functions describing the general cross-section and three-dimensional (3D) boundary, respectively. The curvature rate (β) is quantified by the number (or fraction) of full revolution along the length of the beam. Similarly, if present, the twist rate (α) is quantified by the number of full twists along the length of the beam.

2.1. Formulation of the boundary value problem

The boundary value problem (BVP) governing the dynamics of any structure can be derived using the extended Hamilton's principle,

$$\int_{t_1}^{t_2} \delta H \, \mathrm{d}t = 0, \quad \delta q_i(x, y, z, t) = 0 \quad @ \; t = t_1, \; t_2. \tag{1}$$

Here, $H = T - V + W_{nc}$ is the Hamiltonian term, where T, V, and W_{nc} are the kinetic energy, potential (strain) energy, and the work done by non-conservative forces, respectively; t_1 and t_2 are two instants of time (where the variation is zero); x, y, z are