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A novel approach to study effects of asymmetric stiffness on parametric instabilities of multi-rotor-system

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ABSTRACT

The main focus of this paper is to study effects of asymmetric stiffness on parametric instabilities of multi-rotor-system through extended Lagrangian formalism, where symmetries are broken in terms of the rotor stiffness. The complete insight of dynamic behaviour of multi-rotor-system with asymmetries is evaluated through extension of Lagrangian equation with a case study. In this work, a dynamic mathematical model of a multi-rotor-system through a novel approach of extension of Lagrangian mechanics is developed, where the system is having asymmetries due to varying stiffness. The amplitude and the natural frequency of the rotor are obtained analytically through the proposed methodology. The bond graph modeling technique is used for modeling the asymmetric rotor. Symbol-shakti® software is used for the simulation of the model. The effects of the stiffness of multi-rotor-system on amplitude and frequencies are studied using numerical simulation. Simulation results show a considerable agreement with the theoretical results obtained through extended Lagrangian formalism. It is further shown that amplitude of the rotor increases inversely the stiffness of the rotor up to a certain limit, which is also affirmed theoretically.

1. Introduction

In recent decade, the dynamic behaviour and diagnostics of cracked rotor have gained momentum. At present, many researchers [1,2] are working on cracked rotor system, however very few researchers have focused on the issue of multi-cracked rotor system [3]. In literature, few authors have investigated the issue of crack evaluation for multi-rotor shaft. The investigations and studies on such crack are available in various papers [4,5] for beam structures and [6] for rotor. However, model based identification of cracks in rotor system can be found in paper of Sekhar [7]. Nevertheless, no such paper is available to evaluate the frequency and amplitude of multi-rotor-system, which is being attempted in this paper.

Rotating machinery is generally used in diverse engineering applications, such as industrial turbo-machinery, automobiles, machine tools, household accessories, power plant's turbines and futuristic micro and nano-machines etc. Various studies exposed that the vibration brings a change in the physical properties like mass, stiffness and damping capacity that forces a change in modal parameters like natural frequencies. The existence of any type of asymmetries, transverse cracks,

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Nomenclature

m_1, m_2	Rotor mass of rotor 1 & 2
$K_1(\eta)$, $K_2(\eta)$	Time varying stiffness of rotor 1 & rotor 2
$\Delta K(\eta)$	Time varying asymmetry in stiffness for rotor 1 in x-direction
A_1, $A2$	Amplitude of rotor1 & rotor 2
J	Moment of inertia of the rotor masses
R_i, R_a	Damping coefficient of the rotor 1(internal and external respectively)
R_i', R_a'	Damping coefficient of the rotor 2 (internal and external respectively)
R_c	Resistance of dissipative coupling
$x_i(), y_i()$	Displacements in umbra time/real time, where $i = 1 \ldots. n$
$\dot{x}_i(), \dot{y}_i()$	Velocities in umbra time/real time, where $i = 1 \ldots. n$
$\ddot{x}_i(), \ddot{y}_i()$	Acceleration in real time, where $i = 1 \ldots. n$
$\theta(), \theta'()$	Angular displacements in umbra time/real time in rad
$\dot{\theta}(), \dot{\theta}'()$	Angular velocities in umbra-time/real time in rad sec-1
p( )	Momentum umbra time/real time
q( )	Generalized displacement umbra time/real time
$\dot{q}()$	Generalized velocity umbra time/real time
V^p	p^{th} infinitesimal generator of symmetric group
V_t^p	Real time component of p^{th} infinitesimal generator
V_η^p	Umbra time component of p^{th} infinitesimal generator
p_{x_1}, p_{y_1}	Linear momentum of rotor 1 in x and y direction
p_{x_2}, p_{y_2}	Linear momentum of rotor 2 in x and y direction
p_θ	Angular momentum
L^*	Umbra-Lagrangian of the system
H_i^*, H_e^*	Interior and exterior Umbra-Hamiltonian function
$\dot{\theta}(t)$, $\dot{\theta}'(t)$	Shaft spinning speeds of rotor 1 and rotor 2
ω_1, ω_2	Natural frequency of limiting orbit of rotor1 and rotor 2
t	Real time in sec
η	Umbra time in sec
Ω	Constant excitation frequency in rad sec-1
V_c^*	Umbra-potential for compliance elements
e	Generalized force
T^*	Umbra-kinetic energy
f	Generalized velocity
T_c^*	Umbra-co-kinetic energy
V_R^*	Umbra-potential for resistive elements
V_p^*	Umbra-potential for external forces
V^*	Umbra-potential for external forces
H^*	Umbra-Hamiltonian
F	External force with time fluctuation

Bond graph Symbols

0	Common effort junction
1	Common flow junction
I	I element or Inertial element or Inertance
C	C element or compliant element
R	R element or Dissipative element
SE	Source of effort
SF	Source of flow
GY	Gyrator

notches, slits and any inhomogeneity in parameters in any rotating machine/components may cause sudden and total failure. So, there is a urgent requirement to examine such flaws or defects so that required corrective or preventive measures may be appropriately taken. The presence of any cracks or inhomogeneity in parameters in any machines or structural elements mainly leads to reduction of stiffness, which generally effects the dynamic characteristics of rotating machines. As