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Closed-form eigensolutions of nonviscously, nonproportionally damped systems based on continuous damping sensitivity

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ABSTRACT

In this paper, nonviscous, nonproportional, vibrating structures are considered. Nonviscously damped systems are characterized by dissipative mechanisms which depend on the history of the response velocities via hereditary kernel functions. Solutions of the free motion equation lead to a nonlinear eigenvalue problem involving mass, stiffness and damping matrices. Viscoelasticity leads to a frequency dependence of this latter. In this work, a novel closed-form expression to estimate complex eigenvalues is derived. The key point is to consider the damping model as perturbed by a continuous fictitious parameter. Assuming then the eigensolutions as function of this parameter, the computation of the eigenvalues sensitivity leads to an ordinary differential equation, from whose solution arises the proposed analytical formula. The resulting expression explicitly depends on the viscoelasticity (frequency derivatives of the damping function), the nonproportionality (influence of the modal damping matrix off-diagonal terms). Eigenvectors are obtained using existing methods requiring only the corresponding eigenvalue. The method is validated using a numerical example which compares proposed with exact ones and with those determined from the linear first order approximation in terms of the damping matrix. Frequency response functions are also plotted showing that the proposed approach is valid even for moderately or highly damped systems.

1. Introduction

Nonviscous damping materials are widely used for vibration control within many applications of mechanical, civil and aeronautical engineering. These type of energy dissipation devices can also be known as viscoelastic damping. The physical modeling of vibrating structures under viscoelastic damping results in a complex problem since energy dissipation is characterized by hereditary mechanisms: damping forces are function of the time-history of the velocity response. In mathematical terms, this behavior is represented by convolution integrals involving the degrees-of-freedom (dof) velocities over certain kernel functions. Hence, time-domain response is governed by the following system of linear integro-differential equations

$$\mathbf{M}\ddot{\mathbf{u}} + \int_{-\infty}^{t} \mathcal{G}(t-\tau)\dot{\mathbf{u}}\,\mathrm{d}\tau + \mathbf{K}\mathbf{u} = \mathfrak{f}(t), \quad \mathbf{u}(0) = \mathbf{u}_0, \quad \dot{\mathbf{u}}(0) = \dot{\mathbf{u}}_0 \tag{1}$$

where $\mathbf{u}(t) \in \mathbb{R}^n$ represents the array containing the n dof's, $\mathbf{M} \in \mathbb{R}^{n\times n}$ and $\mathbf{K} \in \mathbb{R}^{n\times n}$ are the mass and stiffness matrices. We assume $\mathbf{M}$ to be positive definite and $\mathbf{K}$ positive semidefinite; $\mathcal{G}(t) \in \mathbb{R}^{n\times n}$ is the viscoelastic damping matrix in the time domain containing the hereditary kernel functions, which must satisfy the necessary conditions given by Golla and Hughes [1] to induce a dissipative behavior. The viscous damping can be considered as a particular case with $\mathcal{G}(t) \equiv \mathbf{C}\,\delta(t)$, where $\mathbf{C}$ is the viscous

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damping matrix and $\delta(t)$ the Dirac's delta function. The time-domain response governed by Eqs. (1) is closely related to the eigensolutions of the associated nonlinear eigenvalue problem [2]. Due to this nonlinearity (induced by a frequency-dependent damping matrix), the search of eigensolutions is in general much more expensive from a computational point o view than that of classical viscous damping. In this paper, our challenge is to deduce closed-form approximations which, on one hand, takes into account the main features of a nonviscously damped system (viscoelasticity and nonproportionality) and, on the other hand, it only requires the computational complexity needed for solving the undamped eigenproblem (natural frequencies and normal modes).

The Laplace transform of the free-motion equation (1) leads to a nonlinear eigenvalue problem in the frequency domain. This nonlinearity arises from the frequency dependency of the viscoelastic function in the Laplace domain, $\mathbf{G}(s) = \mathcal{L}\{\mathcal{G}(t)\}$. In general, the s-dependent functions within the damping matrix $\mathbf{G}(s)$ can be of different nature as long as they satisfy the necessary conditions given by Golla and Hughes [1] to describe a real dissipative motion. However, two viscoelastic models have been traditionally used for practical applications: nonviscous models based on exponential kernels proposed by Biot [3] and those based on the fractional derivatives studied by Bagley and Torvik [4,5].

Several methods to solve the general nonlinear eigenvalue problem exist in the bibliography. Ruhe [6], Yang [7] and Singh [8] proposed methodologies based on the Taylor series expansion of the transcendental matrices combined with Newton's eigenvalue iteration method. Williams and Kennedy [9] obtained numerical solutions using on the parabolic interpolation of the determinant of the eigenvalue problem. Daya and Potier-Ferry [10], Duigou et al. [11] and Boudaoud et al. [12] developed techniques based on the asymptotic perturbation theory to determine complex frequencies and eigenvectors. Voss [13,14] developed two algorithms based on the shift-and-invert Arnoldi's technique and on the Jacobi-Davidson method, respectively. References [15–17] describe how to transform multiple dof systems based on the Biot's model into a into a extended linear system, which can be solved using state-space techniques. For lightly nonproportional systems, Adhikari and Pascual [18,19] published an iterative method based on the first and second order Taylor series expansion of the modal damping function. Lázaro et al. [20] proposed a recursive approach using the fixed-point iteration. References [21,22] exploits the damping parameters as mathematical variables in certain domain obtaining solutions for both proportional and nonproportional systems. In the same direction, Lázaro et al. [23] derived a closed from expression for the complex eigenvalues of frame structures with viscoelastic layers based on fractional derivatives and assuming light nonproportionality. In these works the derivatives of the eigensolutions respect of certain damping parameter play a special role. The generalization of derivatives of eigenvalues and eigenvectors for viscoelastic structures was analyzed by Adhikari [24,25]. Cortés and Elejebarrieta [26,27] used Adhikari's solutions in an recursive numerical approach, valid even for highly damped systems. Li et al. [28,29] proposed a new method for eigensensitivity analysis based on a new form of normalization. Singh [30] has proposed recently a new numerical approach to estimate simultaneously eigenvalues and eigenvectors using a iterative scheme. Lewandowski [31] developed a recursive numerical method using a perturbation parameter, valid for a special type of viscoelastic damper based on fractional derivatives.

In the present paper, a closed-form expression of the complex eigenvalues for nonviscously nonproportionally damped symmetric systems is derived. In the bibliography, numerous methods based on iterative procedures are provided. Those most relevant are described in the previous paragraph. In this work, we appeal the added value of having analytical forms valid for any nonviscous damping model independently on its nature. In fact, our derivations lead to formulas which explicitly depend on the entrees of the modal damping matrix and on its s-derivatives. We find two advantages in our proposal respect to those methods based on iterative schemes: On one hand, we dispose of a mathematical expression which is explicitly expressing how the eigenvalues depend on the damping parameters. And, on the other hand, the only computational requirements are those needed for solving the natural frequencies and the normal modes of the undamped problem. Recently, Lázaro [32] has deduced a closed-form expression valid for nonproportionally viscously symmetric damped systems. The current work generalizes that paper introducing the viscoelasticity. The method is validated through a multiple degrees-of-freedom system with various damping models with different nature, considering two levels of damping. Additionally, we compare the eigenvalues and frequency response functions with those determined using the linear first order approximation proposed by Woodhouse [33].

2. Eigensolutions of nonviscous and nonproportional systems

In general, the set of eigenvalues and eigenvectors of a linear dynamic system contains itself the complete information needed to construct frequency- and time-domain solutions. The free motion equations are obtained from $\mathfrak{f}(t) \equiv \mathbf{0}$ and $\mathbf{u}_0 = \dot{\mathbf{u}}_0 = \mathbf{0}$ in Eq. (1). Checking solutions of the form $\mathbf{u}(t) = \overline{\mathbf{u}}e^{st}$ we obtain

$$\left[s^2\mathbf{M} + s\mathbf{G}(s) + \mathbf{K}\right]\overline{\mathbf{u}} \equiv \mathbf{D}(s)\overline{\mathbf{u}} = \mathbf{0} \tag{2}$$

where $\mathbf{D}(s) \in \mathbb{C}^{n\times n}$ is the dynamic stiffness matrix. The main difference between viscous and nonviscous systems is found in the nature of the solution of Eq. (2). Assuming that there are not repeated eigenvalues, nonviscous systems are characterized by having $m = 2n + r$ eigenvalues arranged as

$$\left\{s_1, \ldots, s_n, s_1^*, \ldots, s_n^*, s_{2n+1}, \ldots, s_{2n+r}\right\} \tag{3}$$

The subset $\{s_j, s_j^*\}$, $1 \le j \le n$ are n pairs of complex-conjugate eigenvalues, under the hypothesis that no overdamped modes exist. The rest $\{s_j\}$, $2n+1 \le j \le m$ are negative real eigenvalues, characteristic of nonviscous damping governed by a Biot's