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Stabilizing and destabilizing effects of damping in non-conservative systems: Some new results

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ABSTRACT

Previous work has amply demonstrated that non-conservative systems can be made unstable by the application of damping. Systems with two neutrally-stable damping levels, whereby the system initially gains stability but later loses stability as the level of damping is increased, have also been observed. The phenomenon of three damping-induced stability transitions has not been reported in the literature. Here we show that the addition of damping can cause non-conservative systems to become stable, then unstable, then stable again at the same value of the non-conservative forcing variable. This combination of stability transitions is found to exist for several example systems, including linkages with follower end forces and fluid-conveying pipes. Another interesting observation is that a given system can exhibit different forms of stability transitions in different regions of its parameter space. In a particular example, the neutral stability curves corresponding to two different modes are observed to intersect, such that the boundary separating the stable and unstable regions is piecewise continuous. This observation requires that the accepted definitions of "stabilizing" and "destabilizing" roles of damping be revised. All of these stability transition behaviors were found by applying the Routh-Hurwitz procedure, whereby the traditional procedure is first applied to the characteristic polynomial of the system, and then again to guarantee the existence of a second-order auxiliary polynomial in the Routh array. This procedure is developed in the context of examples, each of which concerns a classical apparatus who dynamics are more interesting than previously believed.

1. Introduction

A counter-intuitive property of some non-conservative systems is that the application of damping can render the system unstable. This property is frequently referred to as Ziegler's paradox [1] after a classical finding that addition of viscous damping to the joints of a double pendulum subjected to a follower end force destabilizes the system. Since Ziegler's publication in 1952, investigations of the destabilizing effect of damping have continued to the present day [2–8], for example. The effect of both internal (structural) and external (viscous) damping has been investigated in systems subjected to follower end forces; this includes fluid-conveying pipes where the conveyed fluid contributes to an additional damping-like velocity-dependent force in addition to the follower end-force.

Research interest in the general phenomenon of destabilization by damping has been robust. Bottema [9] investigated the conditions for damping to induce instability in Ziegler's work [1] and Bolotin [10] provided general results for the stability

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of two-dof nonconservative systems. Herrmann and Jong [11] established that a relationship exists between the damped and undamped critical loads of a system. Nemat-Nasser and coworkers provided extensions to multi-dof [12] and continuous [13] systems and destabilization by damping in a continuous system was observed experimentally by Gregory and Paidoussis in fluid-conveying pipes [14]. Twenty-one years after Ziegler's finding, the destabilization-by-damping phenomenon was known to be sufficiently common that Done [15] felt compelled to demonstrate mathematically that there exist damping configurations which are never destabilizing. This result was supported by observations published earlier [10,16], and corroborated in other non-conservative systems such as fluid-conveying pipes with different boundary conditions [17], curved pipes [18], tapered columns [19], panel flutter [20], and fluid-conveying shells [21]. Although the destabilizing effect of damping implies divergent behavior of trajectories in the neighborhood of the equilibrium; Kounadis [22] and Luongo et al. [23] used nonlinear analysis to show that the trajectories, despite diverging from the equilibrium, may remain bounded.

Several researchers have provided a physical explanation of the destabilizing effect of damping. Energy methods were used by Semler et al. [3] to investigate discrete systems with viscous damping and continuous systems with Kelvin-Voigt (structural) damping. For discrete systems, it was shown that the phase difference plays an important role in determination of the mode that extracts energy from the follower force and the magnitude of the critical load. For small damping, Gallina [4] used eigenvalue analysis to show that a positive derivative of an eigenvalue with respect to the damping coefficient results in instability. Sugiyama and Langthjem [5] used energy methods to investigate Beck's column and concluded that the follower force can only do work in each cycle if the column vibration has a traveling wave component; additionally, the wave number at the point of application of the force determines the rate of energy growth. Kirillov and Seyranian [24] used perturbation methods to derive necessary and sufficient conditions for the matrix of velocity-dependent forces to guarantee asymptotic stability.

It has also been shown that non-conservative systems may exhibit two stability transitions [3], such that increasing the damping for a given non-conservative force can stabilize an unstable system, which then becomes unstable again as damping is further increased. The appearance of multiple damping-induced stability transitions has received considerably less attention in the literature. Many efforts in the field consider systems subject to levels of damping which are too small to produce the phenomenon [5,6,11,12,24,25], for example, although these levels may be justified by mathematical tractability of the problem or the application of interest. In at least one case [17], it appears that the phenomenon was overlooked by the authors or was simply outside the scope of their interest.

This work provides several results related to stability transitions produced by damping which have not appeared in the literature. We find that at least one additional stability transition is both possible and common, whereby a system which is unstable for zero damping, is first stabilized, then destabilized, and finally restabilized as the damping is increased. The key to finding this result was the application of the Routh-Hurwitz procedure [26] twice, back-to-back: once for the characteristic polynomial and the second time for the polynomial that guarantees the existence of a second-order auxilliary polynomial in the Routh array. The application of the Routh-Hurwitz procedure twice helps determine the number of physically realizable damping coefficients at which the system is marginally stable. This is illustrated with the help of examples, which show various taxonomies of damping-induced stabilization and destabilization behavior such as purely stabilizing (S), purely destabilizing (D), stabilizing-destabilizing (S-D), destabilizing-stabilizing (D-S), and stabilizing-destabilizing-stabilizing (S-D-S). In the process of investigating these taxonomies, one system was found to exhibit intersecting flutter instability curves to which the previously-offered definition of "destabilizing" and "stabilizing" damping [3] could not be applied. An alternate definition is proposed which can be consistently applied to each of our examples along with the notion of "absolute instability curve," which is introduced to deal with intersecting flutter instability curves. The examples in this work were selected to illustrate both the procedure and the general phenomenon of damping-induced multiple stability transitions. Subsequent to the selection of the examples, it was found that stabilizing and destabilization effects of damping in the three-link pendulum with a follower force and two- and three-dof fluid-conveying pipes [27] had not been examined, and we are pleased to present the results for these systems.

This paper is organized as follows. The method for determining physically realizable damping coefficients using the Routh-Hurwitz procedure two times in succession is first explained with the help of the extensively-studied two-link pendulum with follower end force [1,3,9,11,28]. For the first time in the literature, the S-D-S multiple stability transition is shown. The two-dof fluid-conveying pipe is then examined and the S-D-S multiple stability transition is observed again for specific values of mass fraction parameter. The three-link pendulum with follower force and a three-dof fluid-conveying pipe are then analyzed; multiple stability transitions are observed in addition to the latter system exhibiting the aforementioned intersecting flutter instability curves. For the sake of completeness, the effects of both internal and external damping are investigated in the continuous cantilevered fluid-conveying pipe [3,6,13,14,17]; the S-D-S multiple stability transition is observed for the external damping case along with other damping-related stability transition behaviors.

2. Two degree-of-freedom system with a follower force

Consider the two-link system shown in Fig. 1, which is subjected to the follower force P. Constrained to move in the horizontal plane, the two links are comprised of point masses m_1 and m_2, connected by massless rods of equal length ℓ. The rotational joints have torsional stiffness k_1 and k_2 and their damping coefficients are c_1 and c_2. The stability characteristics of the system can be