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Tuned dynamics stabilizes an idealized regenerative axial-torsional model of rotary drilling

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ABSTRACT

We present an exact stability analysis of a dynamical system idealizing rotary drilling. This system comprises lumped parameter axial-torsional modes of the drill-string coupled via the cutting forces and torques. The kinematics of cutting is modeled through a functional description of the cut surface which evolves as per a partial differential equation (PDE). Linearization of this model is straightforward as opposed to the traditional state-dependent delay (SDDE) model and both the approaches result in the same characteristic equation. A systematic study on the key system parameters influencing the stability characteristics reveals that torsional damping is very critical and stable drilling is, in general, not possible in its absence. The stable regime increases as the natural frequency of the axial mode approaches that of the torsional mode and a 1:1 internal resonance leads to a significant improvement in the system stability. Hence, from a practical point of view, a drill-string with 1:1 internal resonance is desirable to avoid vibrations during rotary drilling. For the non-resonant case, axial damping reduces the stable range of operating parameters while for the resonant case, an optimum value of axial damping (equal to the torsional damping) results in the largest stable regime. Interestingly, the resonant (tuned) system has a significant parameter regime corresponding to stable operation even in the absence of damping.

1. Introduction

Drill-strings play an important role in rotary deep-drilling systems. It comprises a series of drill-pipes which transmit power from the rotary table at the top to the bottom hole assembly (BHA) which carries the drill-collars and the drill-bit [1]. Therefore, any failure in the drill-string results in shut down of the entire drilling operation and enormous economic loss to the drilling industry. Self-excited axial, torsional and lateral vibrations in a drill-string lead to dynamic phenomena of bit-bounce, stick-slip and whirling, respectively, which further cause the failure of drill-strings [2,3]. In order to prevent damage to the drill-strings, it is mandatory to understand the effect of operational and system parameters on the stability of these vibration modes. In the current work, we perform a detailed stability analysis of an idealized lumped parameter axial-torsional model of rotary drilling [4] to understand the key parameters affecting the onset of self-excited vibrations.

Drill-string vibrations have been studied extensively using continuum models [5,6,7,8], lumped parameter models [9,10] and finite element models [11,12]. The focus was on the uncoupled and coupled vibration modes and frequencies, failure analysis and understanding the sources of self-excitation (see the review in Ref. [13]). Under the lumped parameter paradigm, the first model for bit-bounce in drill-strings including only the axial mode was proposed by Spanos et al. [14] for the roller cone drill-bit. They

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obtained the relationship between weight-on-bit and the amplitude of the axial bit motion leading to bit lift-off (bit-bounce). Stick-slip in drill-strings was first modeled by Dawson et al. [15] again considering only the torsional mode and an angular velocity dependent bit-rock interaction. They found the critical angular speed for sticking to occur. Following these pioneering studies, different models for velocity dependent bit-rock interaction have been considered to study coupled axial-torsional vibrations leading to stick-slip [16,17].

In 2007, Richard et al. [18] proposed a rate independent bit-rock interaction model (RGD model) to study the stick-slip instability. They considered lumped parameter axial and torsional modes and incorporated the regenerative effect due to the axial motion which was ignored in previous studies. The coupling between these two modes of vibration was realized through cutting forces and torques leading to a coupled state-dependent delay differential equation (SDDDE) governing the system evolution. However, a linear stability analysis of their model revealed that steady drilling is always unstable for all values of operating parameters [19]. Depouhon et al. [19] further classified the instability into slow and fast instabilities depending on the rate of growth of the amplitude of vibrations and concluded that for all practical purposes, the parameters corresponding to the slow instability represent stable steady drilling.

We note that the RGD model neither considered axial stiffness nor any damping in the system due to the drill-string and mud/drilling-fluid interaction which have been identified as important elements to get more accurate results [6,20,21,22]. Noting these shortcomings, the RGD model was modified by Besselink et al. [23] and Nandakumar and Wiercigroch [24] wherein the bit-rock interaction law is the same but the system has an axial stiffness and damping in both axial and torsional directions. Linear stability analysis of the final coupled SDDDE model [24] was carried out by Nandakumar and Wiercigroch [25] in 2013 wherein they obtained the stability curves in the regime of operating parameters for fixed values of system parameters. In contrast to the RGD model, it was found that the modified system remains stable for a choice of operating parameters. Liu et al. [26] incorporated the model of [25] in a finite-element formulation of the drill-string considering axial and torsional degrees of freedom only and obtained the stability diagram in a three-dimensional space of angular velocity, depth of cut and the cutting coefficient.

It is useful to note that SDDDEs are always non-linear because of the appearance of the state in the determination of the delay and "true" linearization is not possible since the solution of the system is not differentiable with respect to the state-dependent delay [27,28,29,30]. As a result, one tries to develop a linear DDE with a constant delay associated with the original system in the sense that they have the same local stability properties [28,29]. Accordingly, the stability analysis of rotary drilling conducted using the SDDDE model is always approximate in nature. Recently, Gupta and Wahi [4] developed a global axial-torsional model to study the dynamics of rotary drilling following the approach of Wahi and Chatterjee [31] for self-interrupted turning. In the present paper, we perform a stability analysis of this global model and systematically study the effect of different system parameters on the stable regime of operating parameters. We note that the linearization of our model is straightforward and hence, the stability analysis is exact. Interestingly, we find that the tuned axial-torsional system (both modes having the same natural frequencies) leads to enhanced stability which is in contrast to the results of [32] for multi-cutter turning. Accordingly, it is desirable to have 1:1 internal resonance between the axial and torsional modes of a drill-string to achieve steady rotary drilling without vibrations.

Rest of the paper is organized as follows. In section 2, we outline the mathematical model (developed in Ref. [4]) of rotary drilling employed for the analysis in the current work. Linearization of this model about the steady drilling state followed by a stability analysis is presented in Section 3. In Section 4, we discuss the stability charts and the effect of system damping and axial stiffness on the stability of steady drilling. Finally, some conclusions are drawn in Section 5.

2. Mathematical formulation

A global model was recently developed by Gupta and Wahi [4] to study the axial-torsional dynamics of rotary drilling. Note that a complete model for rotary drilling comprises of three modules: (i) a model for the dynamics of the drill-strings and the drill-bit, (ii) modeling of the regenerative cutting process and finally (iii) a model for the bit-rock interaction forces. These three modules are assembled together taking into account all inter-relations to get the complete rotary drilling model. The global model in Ref. [4] provided an alternate formulation for the regenerative cutting process which can be coupled with any model for drill-string and drill-bit dynamics, and bit-rock interaction law. In particular [4], considered the drill-string model of [24,25] and the bit-rock interaction law from Ref. [18]. In this paper, we systematically study the stability of steady drilling using this combination of the modules. For completeness of presentation, we briefly outline the key steps in the development of this model.

The drill-string is modeled as a two degree-of-freedom system for the axial and torsional motions using lumped parameters as shown in Fig. 1i. The various lumped parameters are: M as the combined mass of the drill-string and the BHA, K_a as the axial spring stiffness, C_a as the viscous damping coefficient, J as the combined rotary inertia (J) of the pipes and the BHA about the rotational axis, K_t as the torsional spring stiffness and C_t as the torsional viscous damping coefficient. The drill-string rotates with an angular velocity Ω_0. We impose a net weight on the drill-bit (WOB) W_0 by fixing the axial load at the top which results in a constant feed-velocity of V_0 under steady drilling conditions. However, during unsteady drilling involving axial vibrations of the drill-string, both the weight on the drill-bit and the feed velocity V_0 changes since the correct boundary condition at the top surface involves an equivalent spring-mass-damper system for the traveling block and the hoisting cable [5,7,13,14,20]. Incorporation of this boundary condition will necessarily involve at least an additional axial node at the top adding to the complexity of the model. To avoid this complexity, one can either assume a constant force at the top with negligible mass