Journal of Sound and Vibration ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Optimization of directional elastic energy propagation

Erik Andreassen [a,*], Hannah R. Chang [b], Massimo Ruzzene [b], Jakob Søndergaard Jensen [a]

[a] Department of Mechanical Engineering, Technical University of Denmark, Nils Koppels Allé, Building 404, Denmark
[b] School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332, United States

ARTICLE INFO

Article history:
Received 24 June 2015
Received in revised form
21 January 2016
Accepted 4 March 2016
Handling Editor: G. Degrande

Keywords:
Wave propagation
Perforated plate
Microstructure
Topology optimization
Phononic crystal
Group velocity

ABSTRACT

The aim of this paper is to demonstrate how topology optimization can be used to design a periodically perforated plate, in order to obtain a tailored anisotropic group velocity profile. The main method is demonstrated on both low and high frequency bending wave propagation in an aluminum plate, but is general in the sense that it could be used to design periodic structures with frequency dependent group velocity profiles for any kind of elastic wave propagation. With the proposed method the resulting design is manufacturable. Measurements on an optimized design compare excellently with the numerical results.

1. Introduction

Wave propagation in an elastic medium depends on the stiffness and density distribution. The governing equation for linear elastic wave propagation is

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \partial^T \mathbf{C}\, \partial \mathbf{u}, \tag{1}$$

where ρ is the density, $\mathbf{u}$ is the displacement field, t is time, ∂ is the spatial derivative operator, and $\mathbf{C}$ is the constitutive matrix.

By periodic variations in the density or stiffness ($\mathbf{C}$) of the structure, one can achieve unique wave propagation properties, such as bandgaps [1], wave beaming [2], and negative refraction [3–5]. In photonics, where analogous phenomena can be achieved by periodic variations in optical properties, a phenomenon called slow light [6], where the group velocity of a light pulse is lower than the speed of light, has received considerable attention. While Cox and Dobson [7] were some of the first to use topology optimization to design photonic bandgap structures, in [8] it is illustrated how topology optimization [9] can be used to design a photonic crystal with tailored group velocity properties.

Returning to elastic wave propagation; for materials where dissipation is negligible, the group velocity corresponds to the energy velocity [10], and controlling how the energy propagates have several possible applications. A simple and obvious one is vibration shielding, but it could also be used in more advanced applications, such as energy harvesting [11].

* Corresponding author.
E-mail address: erand@mek.dtu.dk (E. Andreassen).

http://dx.doi.org/10.1016/j.jsv.2016.03.002

For a periodic structure the wave solution to the full dynamic problem in Eq. (1) can be expanded according to the Bloch theorem [10]:

$$\mathbf{u}(\mathbf{x},t)=\tilde{\mathbf{u}}(\mathbf{x})e^{i\mathbf{k}^T\mathbf{x}}e^{i\omega t}, \tag{2}$$

where $\tilde{\mathbf{u}}$ is the periodic displacement field, $\mathbf{k}=(k_x,k_y,k_z)^T$ is the wave vector, and ω is the temporal frequency. This allows a characterization of the dynamic properties by analyzing the unit cell. Inserting Eq. (2) into Eq. (1) will yield an eigenvalue problem on the unit cell, which, as shown by the authors in an earlier paper [12], can be used to compute the group velocity $\mathbf{c}_g$ for a given frequency ω.

The aim of this paper is to present a method, based on topology optimization, for tailoring the group velocity profile in a structure by creating a periodic pattern in the structure. The method will be demonstrated with the design of a perforated plate. The goal is to make flexural (transverse) waves propagate faster in one direction.

Focusing on a plate structure, as opposed to a full three-dimensional structure, keeps the computational burden relatively low. Furthermore, it makes an experimental verification simpler, because the flexural wave propagation in a perforated plate can easily be measured using an optical measuring technique, such as a scanning laser vibrometer (SLV). That said, the presented method is also applicable to three-dimensional problems.

As already mentioned, the gradient-based method called topology optimization will be used to tailor the unit cell design. In this paper we present methods to tailor the group velocity profile, while the focus of most of the earlier work in elastic wave propagation in periodic structures has been maximization of bandgaps. The first to apply topology optimization to design periodic structures with elastic wave propagation in mind was Sigmund and Jensen [13]. Halkjær et al. [14,15] used topology optimization to design unit cells for maximum bending wave bandgaps. Similarly, El-Sabbagh et al. [16] maximized the bandgaps for bending wave propagation, but considered a finite plate with a finite periodic pattern. While Larsen et al. [17] optimized the wave propagation pattern in a finite plate, without requiring a periodic pattern in the plate.

We start by presenting the optimization problem in Section 2, where the numerical modelling is also discussed. In Section 3 several optimized designs are presented and discussed. Furthermore, one of the designs has been manufactured by perforating an aluminum plate, and the experimental results from SLV measurements are compared to the numerical predictions in Section 4.

2. Theory

2.1. Group velocity of elastic waves

The derivation of the equations necessary to compute the group velocity profile (for flexural waves in a plate with a periodic pattern) is described in detail in [12], and it will not be repeated here. Numerically, the group velocity profile at a given frequency ω can be computed by first finding the wave vectors by solving the discretized eigenvalue problem

$$\left(\mathbf{K}_k(\theta,\omega)-k\mathbf{M}_k(\theta)\right)\mathbf{u}_k=\mathbf{0}, \tag{3}$$

where θ is the angle between the wave vector components k_x and k_y, ω is the temporal frequency and $k=|\mathbf{k}|$ is the wave vector length, and the finite element matrices $\mathbf{K}_k$ and $\mathbf{M}_k$ for a Mindlin plate problem are given in Appendix A. Fig. 1 provides a basic illustration of the wave vector and its components along with an indication of the irreducible Brillouin zone corresponding to the repetitive unit cell (see e.g. [10] for a fundamental explanation of Brillouin zones for periodic media).

For a given combination of θ and k the group velocity $\mathbf{c}_g=(c_{gx},c_{gy})^T$ is given as

$$c_{gx}=\frac{1}{2\omega}\frac{\mathbf{v}^T\left(-i\mathbf{K}_1+2k\cos(\theta)\mathbf{K}_3+k\sin(\theta)\mathbf{K}_4\right)\mathbf{u}}{\mathbf{v}^T\mathbf{M}\mathbf{u}}, \tag{4}$$

$$c_{gy}=\frac{1}{2\omega}\frac{\mathbf{v}^T\left(-i\mathbf{K}_2+2k\sin(\theta)\mathbf{K}_5+k\cos(\theta)\mathbf{K}_4\right)\mathbf{u}}{\mathbf{v}^T\mathbf{M}\mathbf{u}}, \tag{5}$$

where the computations of the matrices $\mathbf{K}_i$ and $\mathbf{M}$ are detailed in Appendix A. $\mathbf{u}$ can be found from $\mathbf{u}_k$ (the solution to Eq.

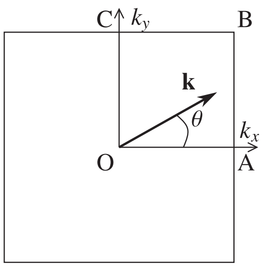

Fig. 1. Illustration of the Brillouin zone and wave vector direction specified by the angle θ.