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High frequency analysis of a plate carrying a concentrated nonlinear spring–mass system

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ABSTRACT

Examining the behavior of dynamical systems with many degrees of freedom undergoing random excitation at high frequency often requires substantial computation. These requirements are even more stringent for nonlinear systems. One approach for describing linear systems, Asymptotic Modal Analysis (AMA), has been extended to nonlinear systems in this paper. A prototypical system, namely a thin plate carrying a concentrated hardening cubic spring–mass, is explored. The study focuses on the response of three principal variables to random, frequency-bounded excitation: the displacement of the mounting location of the discrete spring–mass, the relative displacement of the discrete mass to this mounting location, and the absolute displacement of the discrete mass. The results indicate that extending AMA to nonlinear systems for input frequency bands containing a large number of modes is feasible. Several advantageous properties of nonlinear AMA are found, and an additional reduced frequency-domain modal method, Dominance-Reduced Classical Modal Analysis (DRCMA), is proposed that is intermediate in accuracy and the cost of computation between AMA and Classical Modal Analysis (CMA).

1. Introduction

The modeling of dynamical systems responding with many modes has been a subject of interest for quite some time. Lyon, in 1975, proposed Statistical Energy Analysis (SEA) by assuming equipartition of energy in the active modes in a responding dynamical system [1]. Consequently, in SEA, energy is uniformly distributed through space as well as these modes. SEA developed substantially over the following decades, and stimulated other investigations of alternate modal approaches. Dowell and Kubota laid the foundations of one such method called Asymptotic Modal Analysis (AMA) by describing the high frequency response of a plate undergoing banded, random excitation [2]. The objective of their investigations was to verify the results of SEA by considering the limit of CMA when the number of modes responding in a certain bandwidth becomes large. Transitively, this tests the hypothesis of equipartition of energy.

AMA proved valuable in its own right, as it accurately determined the response of continuous systems experiencing high-frequency random excitation with dramatically lower computational costs. This accuracy even covered "special points", such as the location of an applied point load, where the response is locally greater than that for an arbitrary location of a continuous system. The behavior of these "special points" relative to other locations in a dynamical system was first studied by Crandall [3], whose results were corroborated by Dowell and Kubota in subsequent investigations.

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Nomenclature

A_p	plate area
a_m	modal displacement
D	plate bending stiffness
E	plate material elastic modulus
F	quasi-random time-dependent forcing
F_0	force amplitude of one ergodic input force signal
F_i	time-dependent force for ergodic input force signal i
g	coordinate transfer function summation expression
H_z	relative displacement transfer function
H_{z+z_0}	discrete mass absolute displacement transfer function
H_{z_0}	spring mounting location transfer function
H_λ	constraint force transfer function
h	plate thickness
i	counter for the ergodic input force signal
k	linear spring coefficient
M	coupled mode number
M_0	discrete mass
M_m	modal mass of the plate
M_r	representative AMA Mode number
m	ordered plate mode number
m_x	x-direction plate mode number
m_y	y-direction plate mode number
m_p	plate density per unit area
N_{sig}	total number of ergodic signals used in the study
n_{cycles}	number of cycles studied at steady state during the time-march
q	coordinate representative variable
S_m	characteristic equation
T	timespan of the steady-state simulation window
t_{ss}	time necessary to reach approximate steady-state behavior
x_0	x-coordinate location of the spring mount
x_F	x-coordinate location of the applied force
y_0	y-coordinate location of the spring mount
y_F	y-coordinate location of the applied force
z	displacement from the spring mount location to the discrete mass
$(z+z_0)_i$	absolute displacement of the discrete mass from signal i
Z	relative displacement amplitude
z_0	displacement of the spring mount location
z_{0i}	displacement of the spring mount location from ergodic signal i
z_i	relative displacement from ergodic input force signal i
α	nonlinear spring coefficient
ΔM	number of modes in a bandwidth
$\Delta\omega$	bandwidth
ϵ	settling error
ζ_M	coupled modal damping ratio
ζ_m	plate modal damping ratio
$(\zeta\omega)_c$	damping ratio–natural frequency constant
λ	Lagrange multiplier
ν	plate material Poisson's ratio
ρ	plate material density
Φ_{F_i}	input force power spectrum
ϕ_i	random phase shift for the ergodic input force signal i
ψ_m	mode shape of plate mode m
ω	frequency
ω_0	spring natural frequency
ω_c	input band center frequency
ω_M	natural frequency of coupled mode M
ω_m	natural frequency of plate mode m
ω_{max}	upper bound of the input frequency band
ω_{min}	lower bound of the input frequency band
ω_r	representative AMA frequency

Dowell and Kubota then expanded AMA by studying a plate carrying a concentrated mass [4] and a plate carrying a concentrated spring–mass system [5]. The results of these investigations agreed, again, with the work of Crandall, as the mounting location of the concentrated mass and concentrated spring–mass are found to be "special points" as well. However, the work done by Dowell and Kubota did more than further verify the increased response at certain points in a system; it suggested that AMA can be extended to practical systems. Component coupling among subsystems and nonlinearity must be addressed in order to make this a reality – both of which have proven challenging for SEA as well. Some assert that the issue of coupling between two or more subsystems has been addressed within SEA [6], but it is the contention of the present author that AMA can deal with both of these subjects more rigorously and accurately by consideration of system transfer functions among components. For an informative and well written review of the state of the art for SEA, see the article by Shorter [6].

The present work will modify the system in [5] by adding nonlinearity to the discrete spring–mass element. Consequently, it is strongly suggested that the reader review references [4,5] for the foundations of this investigation. The effect of the nonlinearity will be principally studied, but the work also seeks to gain further insight into coupled systems in both the linear and nonlinear regimes and to establish methods that increase accuracy without accruing substantially greater computational costs.

2. Analysis

The prototypical system in question is a plate carrying an undamped, nonlinear spring–mass system.

The schematic of the system illustrated in Fig. 1 is the example by which AMA is extended to nonlinear systems.

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